Higgs and Supersymmetric Particle Signals
at the Infrared Fixed Point of the Top Quark Mass

M. Carena and C.E.M. Wagner

Theory Division, CERN, CH 1211, Geneva, Switzerland.

Abstract

We study the properties of the Higgs and Supersymmetric particle spectrum, associated with the infrared fixed point solution of the top quark mass in the Minimal Supersymmetric Standard Model. We concentrate on the possible detection of these particles, analysing the deviations from the Standard Model predictions for the leptonic and hadronic variables measured at LEP and for the $b \to s\gamma$ decay rate. We consider the low and moderate $\tan\beta$ regime, imposing the constraints derived from a proper radiative $SU(2)_L \times U(1)_Y$ symmetry breaking and we study both, the cases of universal and non–universal soft supersymmetry breaking parameters at high energies. In the first case, for any given value of the top quark mass, the Higgs and supersymmetric particle spectrum is completely determined as a function of only two soft supersymmetry breaking parameters, implying very definite experimental signatures. In the case of non–universal mass parameters at M_{GUT}, instead, the strong correlations between the sparticle masses are relaxed, allowing a richer structure for the precision data variables. We show, however, that the requirement that the low energy theory proceeds from a grand unified theory with a local symmetry group including $SU(5)$ strongly constrains the set of possible indirect experimental signatures. As a general feature, whenever a significant deviation from the Standard Model value of the precision data parameters is predicted, a light sparticle, visible at LEP2, appears in the model.

1 Introduction

In the present evidence of a heavy top quark, it is of interest to study in greater detail the phenomenological implications of the infrared fixed point predictions for the top quark mass. The low energy fixed point structure of the Renormalization Group (RG) equation of the top quark Yukawa coupling is associated with large values of this coupling at the high energy scale, which, however, remain in the range of validity of perturbation theory [1]. Within the Minimal Supersymmetric Standard Model (MSSM) [2], [3], the infrared fixed point structure determines the value of the top quark mass as a function of $\tan\beta = v_2/v_1$, the ratio of the two Higgs vacuum expectation values. In fact, for a range of high energy values of the top quark Yukawa coupling, such that it can reach its perturbative limit at some scale $M_X = 10^{14} - 10^{19}$ GeV, the value of the physical top quark mass is focused to be $M_t = 190$–$210\,\mathrm{GeV}\sin\beta$, where the variation in M_t is mainly due to a variation in the value of the strong gauge coupling, $\alpha_3(M_Z) = 0.11$–0.13. There is also a small dependence of the infrared fixed point prediction on the supersymmetric spectrum, which, however, comes mainly through the dependence on the spectrum of the running of the strong gauge coupling. Moreover, considering the MSSM with unification of gauge couplings at a grand unification scale M_{GUT} [4], the value of the strong gauge coupling is determined as a function of the electroweak gauge couplings while its dependence on the SUSY spectrum can be characterized by a single effective threshold scale T_{SUSY} [5]-[6]. Thus, the stronger dependence of the infrared fixed point prediction on the SUSY spectrum can be parametrized through T_{SUSY}. There is also an independent effect coming from supersymmetric threshold corrections to the Yukawa coupling, which, for supersymmetric particle masses smaller or of the order of 1 TeV, may change the top quark mass predictions in a few GEV, but without changing the physical picture [7].

The infrared fixed point structure is independent of the particular supersymmetry breaking scheme under consideration. On the contrary, since the Yukawa couplings – especially if they are strong – affect the running of the mass parameters of the theory, once the infrared fixed point structure is present, it plays a decisive role in the resulting (s)particle spectrum of the theory, its predictive power being of course dependent on the number of initial free independent soft SUSY breaking parameters. In addition, to assure a proper breakdown of the electroweak symmetry, one needs to impose conditions on the low energy mass parameters appearing in the scalar potential. Indeed, the condition of a proper radiative $SU(2)_L \times U(1)_Y$ breaking, together with the top quark Yukawa coupling infrared fixed point structure yields interesting correlations among the free high energy mass parameters of the theory, which then translate into interesting predictions for the Supersymmetric (SUSY) spectrum [8] - [16]. Such correlations depend, however, on the exact soft supersymmetry breaking scheme. In particular, in the minimal supergravity model, in which common masses for all the scalars and gaugino masses at the high energy scale are considered, it follows that, once the value of the top quark mass is given, the whole spectrum is determined as a function of

two parameters [8],[9]. In models in which the universality condition for the high energy mass parameters is relaxed, the predictions derived from the infrared fixed point structure are, instead, weaker. Nevertheless, the infrared fixed point structure implies always an effective reduction by two in the number of free parameters of the theory.

The infrared fixed point of the top quark mass is interesting by itself, due to the many interesting properties associated with its behaviour. Moreover, it has been recently observed in the literature that the condition of bottom-tau Yukawa coupling unification in minimal supersymmetric grand unified theories requires large values of the top quark Yukawa coupling at the unification scale [5]-[6], [18]-[21]. Most appealing, in the low and moderate $\tan\beta$ regime, for values of the gauge couplings compatible with recent predictions from LEP and for the experimentally allowed values of the bottom mass, the conditions of gauge and bottom–tau Yukawa coupling unification predict values of the top quark mass within 10% of its infrared fixed point values [5],[22].

In section 2 we concentrate on the infrared fixed point structure of the Yukawa couplings. In section 3 we present the evolution of the mass parameters of the theory in the interesting region of low values of $\tan\beta$, to which we shall restrict ourselves for the present study. Our analysis considers both the case of universal and non–universal boundary conditions for the soft supersymmetry beaking scalar mass parameters at the grand unification scale. In section 4 we investigate the theoretical constraints associated with a proper breakdown of the electroweak symmetry and the requirement of stability of the effective potential by avoiding possible color breaking minima. Complementing the above constraints with the properties of the top quark infrared fixed point structure, we define the allowed low energy mass parameter space as a function of their high energy values. In section 5 we present the results of the above analysis translated into predictions for the Higgs and supersymmetric particle spectra. In section 6, a discusion of the precision data variables to be analysed in the present work is presented. The results for the experimental variables as a function of the supersymmetric spectrum is analysed in section 7. In section 8 we analyse the correlations between the different experimental variables and their phenomenological implications. We reserve section 9 for our conclusions.

2   Infrared Fixed Point Structure

In the Minimal Supersymmetric Standard Model, with unification of gauge couplings at some high energy scale $M_{GUT} \simeq 10^{16}$ GeV, the infrared fixed point structure of the top quark Yukawa coupling may be easily analyzed, in the low and moderate $\tan\beta$ regime ($1 \leq \tan\beta < 10$), for which the effects of the bottom and tau Yukawa couplings are negligible. Indeed, an exact solution for the running top quark Yukawa coupling may be obtained [23],[25] in this regime,

$$Y_t(t) = \frac{2\pi Y_t(0)E(t)}{2\pi + 3Y_t(0)F(t)}, \tag{1}$$

where E and F are functions of the gauge couplings,

$$E = (1 + \beta_3 t)^{16/3b_3}(1 + \beta_2 t)^{3/b_2}(1 + \beta_1 t)^{13/9b_1}, \qquad F = \int_0^t E(t')dt', \tag{2}$$

$Y_t = h_t^2/4\pi$, $\beta_i = \alpha_i(0)b_i/4\pi$, b_i is the beta function coefficient of the gauge coupling α_i and $t = 2\log(M_{GUT}/Q)$. For large values of $\tan\beta$, instead, the bottom Yukawa coupling becomes large and, in general, a numerical study of the coupled equations for the Yukawa couplings becomes necessary even at the one loop level.

For large values of the top quark Yukawa coupling at high energies, Eq. (2) tends to an infrared fixed point value, which is independent of the exact boundary conditions at M_{GUT}, namely,

$$Y_t^{f(Y_t \gg Y_b)}(t) \simeq \frac{2\pi E(t)}{3F(t)}. \tag{3}$$

For values of the grand unification scale $M_{GUT} \simeq 10^{16}$ GeV, the fixed point value, Eq. (3), is given by $Y_t^f \simeq (8/9)\alpha_3(M_Z)$. Since in this case $F(Q = M_Z) \simeq 300$, the infrared fixed point solution is rapidly reached for a wide range of values of $Y_t(0) \simeq 0.1 - 1$. The fixed point structure for the top quark Yukawa coupling implies an infrared fixed point for the running top quark mass, $m_t(t) = h_t(t)v_2 = h_t(t)v\sin\beta$, with $v^2 = v_1^2 + v_2^2$,

$$m_t^{IR}(t) = h_f(t)\ v\ \sin\beta = m_t^{IRmax.}(t)\ \sin\beta, \tag{4}$$

wher we have neglected the slow running of the Higgs vacuum expectation value at low energies. For $\alpha_3(M_Z) = 0.11 - 0.13$, $m_t^{IRmax.}$ is approximately given by

$$m_t^{IRmax.}(M_t) \simeq 196 GeV[1 + 2(\alpha_3(M_Z) - 0.12)]. \tag{5}$$

One should remember that there is a significant quantitative difference between the running top quark mass, and the physical top quark mass M_t, defined as the location of the pole in its two point function. The main source of this difference comes from the QCD corrections, which at the one loop level are given by

$$M_t = m_t(M_t)\left(1 + 4\alpha_3(M_t)/3\pi\right). \tag{6}$$

A numerical two loop RG analysis, shows the stability of the infrared fixed point under higher order loop contributions [21],[6].

A similar exact analytical study can be done for the large $\tan\beta$ regime, when the bottom and top Yukawa couplings are equal at the unification scale, by neglecting in a first

approximation the effects of the tau Yukawa coupling and identifying the right-bottom and right-top hypercharges. The approximate solution for $Y = Y_t \simeq Y_b$ reads [31],

$$Y(t) = \frac{4\pi Y(0)E(t)}{4\pi + 7Y(0)F(t)} \tag{7}$$

Then, if the Yukawa coupling is large at the grand unification scale, at energies of the order of the top quark mass it will develop an infrared fixed point value approximately given by [6], [31],

$$Y_t^{f\ (Y_t=Y_b)}(t) \simeq \frac{4\pi E(t)}{7F(t)} \simeq \frac{6}{7}Y_t^{f(Y_t \gg Y_b)}(t). \tag{8}$$

An approximate expression for the fixed point solution may be found also for values of the bottom Yukawa coupling different from the top quark one [33].

In general, in the large $\tan\beta$ region the bottom quark Yukawa coupling becomes strong and plays an important role in the RG analysis [26]. The are also possible large radiative corrections to the bottom mass coming from loops of supersymmetric particles, which are strongly dependent on the particular spectrum and are extremely important in the analysis, if unification of bottom and tau Yukawa couplings is to be considered [29]–[31]. Moreover, in some of the minimal models of grand unification, large $\tan\beta$ values are in conflict with proton decay constraints [28]. In the special case of tau-bottom-top Yukawa coupling unification the infrared fixed point solution for the top quark mass is not achievable unless a relaxation in the high energy boundary conditions of the mass parameters of the theory is arranged, and it is necessarily associated with a heavy supersymmetric spectrum [32]. In the following we shall concentrate on the low and moderate $\tan\beta$ region.

3   Evolution of the Mass Parameters

In this work we shall consider soft supersymmetry breaking mass terms for all the scalars and gauginos of the theory, as well as trilinear and bilinear couplings A_i (with i= leptons, up quarks and down quarks) and B in the full scalar potential, which are proportional to the trilinear and bilinear terms appearing in the superpotential. In the framework of minimal supergravity the soft supersymmetry breaking parameters are universal at the grand unification scale. This implies common soft supersymmetry breaking mass terms m_0 and $M_{1/2}$ for the scalar and gaugino sectors of the theory, respectively, and a common value A_0 (B_0) for all trilinear (bilinear) couplings A_i (B). In addition, the supersymmetric Higgs mass parameter μ appearing in the superpotential takes a value μ_0 at the grand unification scale M_{GUT}. In the present work we shall consider a more general case, in which the condition of universality of the soft supersymmetry breaking scalar mass parameters is relaxed. We shall, however, asume that SU(5) is a subgroup of the grand unification symmetry group and, hence, we shall

keep the relations between the soft supersymmetry breaking mass parameters that preserve the SU(5) symmetry. This implies a common gaugino soft supersymmetry breaking mass parameter, common values for the soft supersymmetry breaking parameters of the right and left handed scalar top quarks, but free, independent values for the two Higgs mass parameters at M_{GUT}. The relevance of non–universal soft supersymmetry breaking parameters for the spectrum of the theory in the low $\tan\beta$ regime has been recently emphasized in several works [34]. For definiteness, we shall identify all squark and slepton mass parameters with the ones of the stop quark ones. This requirement has little influence in our analyisis, which mainly depends on the Higgs, chargino and stop spectra.

Knowing the values of the mass parameters at the unification scale, their low energy values may be specified by their renormalization group evolution [23]-[27], which contains also a dependence on the gauge and Yukawa couplings. In particular, in the low and moderate $\tan\beta$ regime, in which the effects of the bottom and tau Yukawa couplings are negligible, it is possible to determine the evolution of the soft supersymmetry breaking mass parameters of the model as a function of their high energy boundary conditions and the value of the top quark Yukawa coupling at M_{GUT}, $Y_t(0)$. Indeed, using Eq. (3) and renaming $Y_t^{f(Y_t \gg Y_b)} = Y_f(t)$, it follows,

$$\frac{6Y_t(0)F(t)}{4\pi} = \frac{Y_t(t)/Y_f(t)}{1 - Y_t(t)/Y_f(t)} \ , \tag{9}$$

with $Y_t/Y_f = h_t^2/h_f^2$ the ratio of Yukawa couplings squared at low energies. The above equation permits to express the boundary condition of the top quark Yukawa coupling as a function of the gauge couplings (through F) and the ratio Y_t/Y_f [23]-[27] giving definite predictions for the low energy mass parameters of the model in the limit $h_t \to h_f$ [8].

Thus, considering the limit of small $\tan\beta$, $\tan\beta < 10$, the following approximate analytical solutions are obtained for the case of non–universal parameters at M_{GUT},

$$m_L^2 \ = \ m_L^2(0) + 0.52M_{1/2}^2 \qquad\qquad m_E^2 = m_E^2(0) + 0.15M_{1/2}^2$$

$$m_{Q(1,2)}^2 \ = \ m_{Q(1,2)}^2(0) + 7.2M_{1/2}^2 \qquad\qquad m_{U(1,2)}^2 \simeq m_{U(1,2)}^2(0) + 6.7M_{1/2}^2$$

$$m_D^2 \ \simeq \ m_D^2(0) + 6.7M_{1/2}^2$$

$$m_Q^2 \ = \ 7.2M_{1/2}^2 + m_Q^2(0) + \frac{\Delta m^2}{3}$$

$$m_U^2 \ = \ 6.7M_{1/2}^2 + m_U^2(0) + 2\frac{\Delta m^2}{3} \tag{10}$$

where E, D and U are the right handed leptons, down-squarks and up-squarks, respectively, L and Q = (T B)^T are the lepton and top-bottom left handed doublets and m_η^2, with

$\eta = E, D, U, L, Q$ are the corresponding soft supersymmetry breaking mass parameters. The subindices (1,2) are to distinguish the first and second generations from the third one, whose mass parameters receive the top quark Yukawa coupling contribution to their renormalization group evolution, singled out in the Δm^2 term,

$$
\begin{aligned}
\Delta m^2 &= -\frac{\left(m_{H_2}^2(0) + m_U^2(0) + m_Q^2(0)\right)}{2}\frac{Y_t}{Y_f} + 2.3A_0M_{1/2}\frac{Y_t}{Y_f}\left(1 - \frac{Y_t}{Y_f}\right) \\
&\quad - \frac{A_0^2}{2}\frac{Y_t}{Y_f}\left(1 - \frac{Y_t}{Y_f}\right) + M_{1/2}^2\left[-7\frac{Y_t}{Y_f} + 3\left(\frac{Y_t}{Y_f}\right)^2\right] .
\end{aligned}
\tag{11}
$$

For the Higgs sector, the mass parameters involved are

$$
m_{H_1}^2 = m_{H_1}^2(0) + 0.52M_{1/2}^2 \quad and \quad m_{H_2}^2 = m_{H_2}^2(0) + 0.52M_{1/2}^2 + \Delta m^2 ,
\tag{12}
$$

which are the soft supersymmetry breaking parts of the mass parameters m_1^2 and m_2^2 appearing in the Higgs scalar potential (see section 4). Moreover, the renormalization group evolution for the supersymmetric mass parameter μ reads,

$$
\mu^2 \simeq 2\mu_0^2\left(1 - \frac{Y_t}{Y_f}\right)^{1/2} ,
\tag{13}
$$

while the running of the soft supersymmetry breaking bilinear and trilinear couplings gives,

$$
B = B_0 - \frac{A_0}{2}\frac{Y_t}{Y_f} + M_{1/2}\left(1.2\frac{Y_t}{Y_f} - 0.6\right).
\tag{14}
$$

$$
A_t = A_0\left(1 - \frac{Y_t}{Y_f}\right) - M_{1/2}\left(4.2 - 2.1\frac{Y_t}{Y_f}\right),
\tag{15}
$$

respectively. Eq. (13) shows that the RG evolution of the supersymmetric mass parameter μ, appearing in the superpotential. Observe that μ formally vanishes at low energies in the limit $Y_t \to Y_f$. However, since $\mu \simeq \sqrt{2}\mu_0 \left(1 - Y_t/Y_f\right)^{1/4}$, μ stays of order μ_0 even for values all values of Y_t within the range of validity of perturbation theory at high energies, $Y_t(0) \leq 1$ $(Y_t/Y_f \leq 0.995)$ [8]. The coefficients characterizing the dependence of the mass parameters on the universal gaugino mass $M_{1/2}$ depend on the exact value of the strong gauge couplings. In the above, we have taken the values of the coefficients that are obtained for $\alpha_3(M_Z) \simeq 0.12$. The above analytical solutions are sufficiently accurate for the purpose of understanding the properties of the mass parameters in the limit $Y_t \to Y_f$.

4   Constraints on the Fixed Point Solutions

The solutions for the mass parameters may be strongly constrained by experimental and theoretical restrictions. The experimental contraints come from the present lower bounds

on the supersymmetric particle masses [35]. Concerning the theoretical constraints, many of them impose bounds on the allowed space for the soft supersymmetry breaking parameters in model dependent ways to various degrees. The conditions of stability of the effective potential and a proper breaking of the $SU(2)_L \times U(1)_Y$ symmetry are, instead, basic necessary requirements, which, complemented with the properties derived from the infrared fixed point structure, yield robust correlations among the free parameters of the theory.

4.1 Radiative electroweak symmetry breaking

The Higgs potential of the Minimal Supersymmetric Standard Model may be written as [3], [36]-[38]

$$
\begin{aligned}
V_{eff} &= m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - m_3^2 (H_1^T i\tau_2 H_2 + h.c.) \\
&+ \frac{\lambda_1}{2} \left(H_1^\dagger H_1 \right)^2 + \frac{\lambda_2}{2} \left(H_2^\dagger H_2 \right)^2 + \lambda_3 \left(H_1^\dagger H_1 \right) \left(H_2^\dagger H_2 \right) + \lambda_4 \left| H_2^\dagger i\tau_2 H_1^* \right|^2,
\end{aligned}
\tag{16}
$$

with $m_i^2 = \mu^2 + m_{H_i}^2$, $i = 1, 2$, and $m_3^2 = B|\mu|$ and where at scales at which the theory is supersymmetric the running quartic couplings λ_j, with $j = 1 - 4$, must satisfy the following conditions:

$$
\lambda_1 = \lambda_2 = \frac{g_1^2 + g_2^2}{4} = \frac{M_Z^2}{2\,v^2}, \qquad \lambda_3 = \frac{g_2^2 - g_1^2}{4}, \qquad \lambda_4 = -\frac{g_2^2}{2} = \frac{M_W^2}{v^2}.
\tag{17}
$$

Hence, in order to obtain the low energy values of the quartic couplings, they must be evolved using the appropriate renormalization group equations, as was explained in Refs. [36]-[39]. The mass parameters m_i^2, with $i = $ 1-3 must also be evolved in a consistent way and their RG equations may be found in the literature [23]-[25],[40],[41]. The minimization conditions $\partial V / \partial H_i |_{<H_i>=v_i} = 0$, which are necessary to impose the proper breakdown of the electroweak symmetry, read

$$
\sin(2\beta) = \frac{2m_3^2}{m_A^2}
\tag{18}
$$

$$
\tan^2 \beta = \frac{m_1^2 + \lambda_2 v^2 + (\lambda_1 - \lambda_2)\, v_1^2}{m_2^2 + \lambda_2 v^2},
\tag{19}
$$

where m_A is the CP-odd Higgs mass,

$$
m_A^2 = m_1^2 + m_2^2 + \lambda_1 v_1^2 + \lambda_2 v_2^2 + (\lambda_3 + \lambda_4)\, v^2.
\tag{20}
$$

Considering the case of negligible stop mixing, and in the low $\tan\beta$ regime, the radiative corrections to the quartic couplings λ_i, with $i = 1, 3$ are small, while $\Delta\lambda_2 = (3/8\pi^2) h_t^4 \ln(m_{\tilde{t}}^2 / m_t^2)$. In this case, the minimization condition Eq. (19), can be rewritten as [36]:

$$
\tan^2 \beta = \frac{m_1^2 + M_Z^2 / 2}{m_2^2 + M_Z^2 / 2 + \Delta\lambda_2 v_2^2}.
\tag{21}
$$

Considering the minimization condition, Eq. (19) and the approximate analytical expressions for the mass parameters m_i, Eq. (12), the supersymmetric mass parameter μ is determined as a function of the other free parameters of the theory,

$$
\begin{aligned}
\mu^2 &= \frac{1}{\tan^2 \beta - 1} \left(m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta - \Delta\lambda_2 v_2^2 \tan^2 \beta \right) \\
&= \mathcal{F}(m_{H_1}(0), m_{H_2}(0), m_Q(0), m_U(0), M_{1/2}, A_0, \tan\beta, Y_t/Y_f).
\end{aligned}
\tag{22}
$$

A somewhat more complicated expression is obtained for the case of mixing in the stop sector [38]. The other minimization condition, Eq. (18) also puts restrictions on the soft supersymmetry breaking parameters. It determines the value of the parameter $\delta = B_0 - A_0$ as a function of the other parameters of the theory [8]. However, as we shall show below, at the fixed point solution the mass parameter δ is not directly related to the range of possible mass values of the Higgs and supersymmetric particles.

4.2   Properties of the Fixed Point Solution.

The ratio of the top quark Yukawa coupling to its infrared fixed point value may be expressed as a function of the top quark mass and the angle β,

$$
\frac{Y_t}{Y_f} = \left(\frac{m_t}{m_t^{IRmax}} \right)^2 \frac{1}{\sin^2 \beta},
\tag{23}
$$

where the exact value of $m_t^{IRmax.}$, Eq. (5), depends on the value of the strong gauge coupling considered, and for the experimentally allowed range it varies approximately between 190–200 GeV. Depending on the precise value of the running top quark mass m_t and $\tan \beta$, the above equation gives a measure of the proximity to the infrared fixed point solution. In the limit $Y_t \to Y_f$, the strong correlation between the top quark mass and the value of $\tan \beta$, Eq.(4), allows to reduce by one the number of free parameters of the theory.

Moreover, at the infrared fixed point, the expressions for the low energy parameters, Eqs. (10)-(15), show that the term Δm^2 and hence the mass parameters $m_{H_2}^2$, m_Q^2 and m_U^2 become very weakly dependent on the supersymmetry breaking parameter A_0. In fact, the dependence on A_0 vanishes in the formal limit $Y_t \to Y_f$ [8]. The only relevant dependence on A_0 enters through the mass parameter B. Therefore, at the infrared fixed point, there is an effective reduction in the number of free independent soft supersymmetry breaking parameters. In fact, the dependence on B_0 and A_0 of the low energy solutions is effectively replaced by a dependence on the parameter $\delta = B_0 - A_0/2$. Since B is not involved in the RG evolution of the (s)particle masses and the squark and slepton mixing for sparticles other than the top squark is very small, the above implies that at the infrared fixed point the dependence of the Higgs and supersymmetric spectrum on the parameter A_0 is negligible [8]. Hence, the infrared fixed point structure translates in a net reduction by two in the number

of free parameters which are relevant in determining the spectrum of the theory. There is also a very interesting behaviour of the low energy mass parameter combination

$$M_{UQ}^2 = m_Q^2 + m_U^2 + m_{H_2}^2 \tag{24}$$

at the infrared fixed point. Indeed, the dependence of M_{UQ} on its high energy boundary condition, $M_{UQ}^2(0) = m_Q^2(0) + m_U^2(0) + m_{H_2}^2(0)$ vanishes in the formal limit $Y_t \to Y_f$. It follows that the infrared fixed point structure of the top quark Yukawa coupling yields an infrared fixed point for the soft supersymmetry breaking parameter A_t as well as for the combination M_{UQ}^2.

Summarizing, for a given value of the physical top quark mass, the running top quark mass is fixed and then at the infared fixed point Eq. (23) fixes $\sin \beta$. Due to the strong correlation of the top quark mass with $\tan \beta$ and the independence of the spectrum on the parameter A_0, for a given top quark mass the Higgs and supersymmetric particle spectrum is completely determined as a function of only the high energy boundary conditions for the scalar and gaugino mass mass parameters. It is then possible to perform a scanning of all the possible values for $m_Q(0)$ ($m_Q(0) \equiv m_U(0)$), $m_{H_1}(0)$, $m_{H_2}(0)$ and $M_{1/2}$, bounding the squark masses to be, for example, below 1 TeV, and the whole allowed parameter space for the Higgs and superparticle masses may be studied. In the following we shall study different boundary conditions for the soft supersymmetry breaking mass parameters, concentrating on those which may yield interesting features for the low energy spectrum. In particular, we shall also consider the case in which all soft supersymmetry breaking scalar masses acquire a common value at the high energy scale, which gives an extremely predictive framework with only two parameters determining the whole Higgs and supersymmetric spectrum.

4.3   Color breaking minima

There are several conditions which need to be fulfilled to ensure the stability of the electroweak symmetry breaking vacuum. In particular, one should check that no charge or color breaking minima are induced at low energies. A well known condition for the absence of color breaking minima is given by the relation [42]

$$A_t^2 \leq 3(M_{UQ}^2) + 3\mu^2. \tag{25}$$

At the fixed point, however, since $A_t \simeq -2.1 M_{1/2}$ and $M_{UQ}^2 \simeq 6 M_{1/2}^2$, this relation is trivially fulfilled [45], [9]. For values of $\tan \beta$ close to one, large values of μ are induced and a more appropriate relation is obtained by looking for possible color breaking minima in the direction $\langle H_2 \rangle \simeq \langle H_1 \rangle$ and $\langle Q \rangle \simeq \langle U \rangle$ [44],[45]. The requirement of stability of the physically acceptable vacuum implies the following sufficient condition [9],

$$(A_t - \mu)^2 \leq 2 \left(m_Q^2 + m_U^2 \right) + \tilde{m}_{12}^2 \tag{26}$$

10

where $\tilde{m}_{12}^2 = (m_1^2 + m_2^2)(\tan\beta - 1)^2/(\tan\beta^2 + 1)$.

If Eq.(26) is not fulfilled, a second sufficient condition is given by

$$\left[(A_t - \mu)^2 - 2\left(m_Q^2 + m_U^2\right) - \tilde{m}_{12}^2\right]^2 \leq 8\left(m_Q^2 + m_U^2\right)\tilde{m}_{12}^2. \tag{27}$$

The above relations, Eqs. (26) and (27) are sufficient conditions since they assure that a color breaking minima lower than the trivial minima does not develop in the theory. If the above conditions are violated, a necessary condition to avoid the existence of a color breaking minima lower than the physically acceptable one is given by

$$V_{col} \geq V_{ph} \tag{28}$$

with

$$V_{col} = \frac{(A_t - \mu)^2 \alpha_{min}^2}{h_t^2(2\alpha_{min}^2 + 1)^3}\left[(m_Q^2 + m_U^2) - 2\tilde{m}_{12}^2\alpha_{min}^4\right], \tag{29}$$

$$V_{ph} = -\frac{M_Z^4}{2(g_1^2 + g_2^2)}\cos^2(2\beta), \tag{30}$$

and

$$\alpha_{min}^2 = \left[(A_t - \mu)^2 - 2(m_Q^2 + m_U^2) - \tilde{m}_{12}^2\right]/(4\tilde{m}_{12}^2). \tag{31}$$

For some of the non-universal conditions one may consider, the right handed stop super-symmetry breaking mass parameter m_U^2 can get negative values. In this case a color breaking minimum may develop in the direction $\langle U \rangle \neq 0$. The value of the tree level potential at this minimum would be

$$V_U = -\frac{9}{8g_1^2}m_U^4, \qquad (m_U^2 < 0) \tag{32}$$

which should be higher than V_{ph} in order to avoid an unacceptable vacuum state. For low values of $\tan\beta \leq 1.5$, the range of parameters leading to a negative value of m_U^2, are automatically excluded either because they lead to values of the lightest CP-even Higgs mass which are experimentally excluded (particularly for $\mu < 0$) or because they lead to tachyons in the stop sector or are in conflict with the absence of color breaking minima in the other directions analysed before, Eqs. (26)–(28). For larger values of $\tan\beta$ and negative values of μ, for which the mixing in the stop sector is small, the requirement $V_U \geq V_{ph}$, with V_U given in Eq. (32), becomes, however, relevant.

5   Higgs and Supersymmetric Particle Spectrum

As we mentioned before, for low values of $\tan\beta$ and for a given value of the top quark mass, the whole spectrum is determined as a function of the free independent soft supersymmetry breaking parameters, $m_Q(0)$ (where $m_Q(0) = m_{\tilde{q}}(0) = m_{\tilde{l}}(0)$), $m_{H_1}(0)$, $m_{H_2}(0)$ and $M_{1/2}$.

Summarizing the results for the relevant low energy mass parameters at the fixed point solution we have:

$$
\begin{aligned}
m_{H_2}^2 &\simeq \frac{m_{H_2}^2(0)}{2} - m_Q^2(0) - 3.5 M_{1/2}^2 \qquad\qquad m_{H_1}^2 \simeq m_{H_1}^2(0) + 0.5 M_{1/2}^2 \\
m_Q^2 &\simeq \frac{2 m_Q^2(0)}{3} - \frac{m_{H_2}^2(0)}{6} + 6 M_{1/2}^2 \\
m_U^2 &\simeq \frac{m_Q^2(0)}{3} - \frac{m_{H_2}^2(0)}{3} + 4 M_{1/2}^2 \\
A_t &\simeq -2.1 M_{1/2} \\
\mu^2 &\simeq \left[m_{H_1}^2(0) + \left(\frac{2 m_Q^2(0) - m_{H_2}^2(0)}{2} \right) \tan^2 \beta \right. \\
&\quad + \left. M_{1/2}^2 \left(0.5 + 3.5 \tan^2 \beta \right) \right] \frac{1}{\tan^2 \beta - 1}
\end{aligned}
\tag{33}
$$

In the following, we shall analyze the contributions of possible light supersymmetric particles to the hadronic and leptonic variables measured at LEP. In fact, concerning indirect searches at LEP, the existence of light charginos and stops may yield interesting supersymmetric signals. The Higgs sector is of course very interesting in itself and it also plays an important role in deriving constraints on the soft supersymmetry breaking parameters, which then translate into restrictions for the stop chargino sectors as well. The dependence of the properties of the spectrum on the high energy boundary conditions for the soft supersymmetry breaking parameters is very important and we shall consider different interesting possibilities in a detailed way. In Table 1 we display the dominant dependence of the low energy scalar mass parameters on their high energy values for the three characteristic soft supersymmetry breaking schemes we shall analyse in this work. The case of universal soft supersymmetry breaking parameters, in which all the soft supersymmetry breaking scalar masses acquire a common value, say m_0, is the most predictive one. In such limit, for a given value of the top quark mass, the whole Higgs and supersymmetric particle spectrum is determined as a function of only two parameters, m_0 and the common gaugino mass $M_{1/2}$ [8].

Another interesting case is the one in which the dependence of μ^2 on the soft supersymmetry breaking parameters of the scalar fields vanishes, implying smaller values for the supersymmetric mass parameter and hence a stronger Higgsino component of the light chargino than in the universal case. This situation follows, in a $\tan \beta$ independent way, if $m_{H_1}(0) = 0$ and $m_{H_2}^2(0) = 2 m_Q^2(0)$. As can be seen in Table 1, the parameter m_U^2 can be render small or negative by increasing m_0, what increases the right handed component of the lightest stop with respect to the one in the case of universal soft supersymmetry breaking parameters at M_{GUT}. As we shall discuss below, a larger Higgsino (right stop) component of the lightest chargino (stop) implies an increase in the supersymmetric Z^0–$b\bar{b}$ vertex corrections.

A non–universal condition for the scalar soft supersymmetry breaking mass parameters can also yield larger values for the stop mass parameters. This situation may be achieved if, for example, we invert the relations used above for $m^2_{H_1}(0)$ and $m^2_{H_2}(0)$, that is to say, $m^2_{H_1}(0) = 2m^2_Q(0)$ and $m^2_{H_2}(0) = 0$. Then, parametrizing the scalar masses as a function of $m_{H_1}(0)$, the values of μ^2 and $m^2_Q + m^2_U$ have the same functional dependence on $m^2_{H_1}(0)$ as the one as a function of m^2_0 in the universal case. However, both m^2_Q and m^2_U increase with $m^2_{H_1}(0)$, breaking the strong correlation present in the universal case between the lightest stop and the gaugino masses [8], [34]. Indeed, even for light charginos, large values of the lightest stop mass may be obtained in this case by taking large values for the scalar mass parameters at the grand unification scale.

Conditions at M_{GUT}	m^2_Q	m^2_U	$m^2_{H_2}$	$m^2_{H_1}$
Universal m^2_0	$\dfrac{m^2_0}{2}$	0	$-\dfrac{m^2_0}{2}$	m^2_0
Case I: $m^2_{H_1}(0) = 0$, $m^2_{H_2}(0) = 2m^2_Q(0)$ $m^2_{H_2}(0) = m^2_0$	$\dfrac{m^2_0}{6}$	$-\dfrac{m^2_0}{6}$	0	0
Case II: $m^2_{H_2}(0) = 0$, $m^2_{H_1}(0) = 2m^2_Q(0)$ $m^2_{H_1}(0) = m^2_0$	$\dfrac{m^2_0}{3}$	$\dfrac{m^2_0}{6}$	$-\dfrac{m^2_0}{2}$	m^2_0

Table 1. Dominant dependence of the low energy soft supersymmetry breaking parameters on their values at the grand unification scale, for a top quark mass at its infrared fixed point value.

The value of the supersymmetric mass parameter μ may be obtained from the other mass parameters through the condition of a proper radiative electroweak symmetry breaking, Eqs. (19), (33). In the following, we shall analyze these three possibilities considering cases I and II as characteristic ones for the study of the possible implications of the deviations from the universal boundary conditions in the soft supersymmetry breaking parameters associated with the scalar sector.

5.1 Stop and Chargino Sectors

Due to the large values of the mass parameter μ at the infrared fixed point for low values of $\tan\beta$, there is small mixing in the chargino and neutralino sectors. Hence, to a good approximation the lightest chargino mass and the lightest and next to lightest neutralino masses are given by $m_{\tilde{\chi}_l^\pm} \simeq m_{\tilde{\chi}_2^0} \simeq 2m_{\tilde{\chi}_1^0} \simeq 0.8M_{1/2}$. This approximate dependence becomes more accurate when larger values of $M_{1/2}$ are considered. For low values of $M_{1/2}$, although the lightest chargino is still mainly a wino, it follows that for positive values of μ, slightly larger values of $M_{1/2}$ are necessary to get a light chargino, with mass close to their production threshold at the Z^0 peak, than those required for negative values of μ.

Concerning the restrictions on the parameter space, more interesting than the chargino sector becomes the stop sector, for which these large values of μ may render the physical squared stop mass negative or too small to be consistent with the present experimental bounds, which we shall take to be $m_{\tilde{t}} > 45$ GeV. The stop mass matrix is given by,

$$M_{\tilde{t}}^2 = \left[\begin{array}{cc} m_Q^2 + m_t^2 + D_{t_L} & m_t(A_t - \mu/\tan\beta) \\ m_t(A_t - \mu/\tan\beta) & m_U^2 + m_t^2 + D_{t_R} \end{array} \right] \tag{34}$$

where $D_{t_L} \simeq -0.35M_Z^2|cos2\beta|$ and $D_{t_R} \simeq -0.15M_Z^2|cos2\beta|$ are the D-term contributions to the left and right handed stops, respectively. The above mass matrix, after diagonalization, leads to the two stop mass eigenvalues, $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$. At the infrared fixed point, the values of the parameters involved in the mass matrix are given in Eq. (33). For values of $\tan\beta$ close to one, the off-diagonal term contribution will be enhanced due to the large values of μ associated with such low values of $\tan\beta$ and, consequently, the mixing may be sufficiently large to yield a tachyonic solution [8]-[17]. Thus, depending on the case considered for the soft supersymmetry breaking scalar masses and its hierarchy with respect to $M_{1/2}$, as well as on the sign of μ, important constraints on the parameter space may be obtained.

For example, if we consider first the universal case with a common scalar mass m_0, for $\tan\beta = 1.2$, which implies $M_t \simeq 160$ GeV and for which the value of the supersymmetric mass parameter $\mu^2 \simeq 4m_0^2 + 12M_{1/2}^2$, it is straighforward to show that, if one considers the regime $M_{1/2}^2 \ll m_0^2$ for both signs of μ (or $M_{1/2}^2 > m_0^2$ for $\mu > 0$), then a tachyon state will develop unless $M_{1/2} \geq m_t$. For $M_{1/2}^2 > m_0^2$ and $\mu < 0$, since there is a partial cancellation of the off-diagonal term which suppresses the mixing, no tachyonic solution may develop and, hence, no constraint is derived from these considerations. However, as we shall show below, restrictions coming from the Higgs sector will constrain this region of parameter space as well. Observe that for these low values of $\tan\beta$, the necessary and sufficient conditions to avoid a color breaking minima, Eqs. (26), (27) and (28), put strong restrictions on the solutions with large left–right stop mixing. For slightly larger values of $\tan\beta \simeq 1.8$, which correspond to much larger values of the top quark mass, $M_t \simeq 180GeV$, the value of $\mu^2 \simeq 1.2m_0^2 + 5.3M_{1/2}^2$ is sufficiently small so that, helped by the factor $1/\tan\beta$ appearing

in the off–diagonal terms in Eq.(34), there is no possibility for a tachyon to develop in this case and, hence, no constraints on $M_{1/2}$ are obtained. Of course, this result holds for larger values of $\tan\beta$ as well. It is interesting to notice that, although there is no necessity to be concerned about tachyons for values of $\tan\beta \simeq 1.8$, it is still possible to have light stops, $m_{\tilde{t}_1} < 150$ GeV, if the value of $M_{1/2} \le 100$ GeV. For larger values of $\tan\beta$ ($M_t > 185$ GeV) a light stop is not possible any longer in the universal case.

Figure 1 shows the dependence of the stop mass on the chargino mass in the case of universal scalar masses at M_{GUT}, for four different values of the top quark mass. For low values of $M_t \le 165$ GeV, the color breaking constraints forbids large mixing in the stop sector and, due to the behaviour of $m_U^2 \simeq 4M_{1/2}^2$, a strong correlation between the lightest stop and the lightest chargino is observed. For larger values of M_t, larger mixing is allowed and a clear distinction between the two signs of μ is observed. This distinction is particularly clear for $M_t \simeq 175$ GeV ($\tan\beta \simeq 1.5$) and disappears for larger values of $\tan\beta$. Observe that just for the interesting region 165 GeV $\le M_t \le 185$ GeV both light stops and light charginos ($m_{\tilde{\chi}_1} < 70$ GeV) are allowed. For larger (lower) values of M_t, it becomes more difficult to get light stops (charginos). Light stops and charginos are very interesting, both for direct experimental searches [43] as well as for indirect searches through deviations from the Standard Model predictions for the leptonic and hadronic variables measured at LEP (see below).

If we consider the non–universal case with $m_{H_1}^2(0) = 0$ and $m_{H_2}^2(0)/2 = m_Q^2(0) = m_0^2/2$ (case I), then if $M_{1/2}$ dominates the supersymmetry breaking, the constraints coming from the requirement of avoiding a very small stop mass are equivalent to the ones obtained in the case of universal mass parameters. If $M_{1/2}$ is much smaller than the parameter m_0, instead, an upper bound on the scalar mass parameter is obtained, $m_0^2 < 6m_t^2$. More generally, in the regime of large values of m_0 and moderate values of $M_{1/2}$, it follows that for positive values of μ, in order to avoid tachyons,

$$m_t^2 > 0.5\left[KM_{1/2}^2 + \sqrt{\left(KM_{1/2}^2\right)^2 - 96M_{1/2}^4 + \left(\frac{m_0^2}{3}\right)^2 + \frac{4}{3}m_0^2M_{1/2}^2}\right] \tag{35}$$

with $K \simeq 10, 4.5, 0.8$ for $M_t \simeq 165, 175, 185$ GeV ($\tan\beta \simeq 1.3, 1.5, 1.9$). For $M_t \le 160$ GeV ($\tan\beta \le 1.2$) this condition cannot be fulfilled since for values of $M_{1/2}$ consistent with the present experimental bounds on the chargino and gluino masses, already the $M_{1/2}$ dependent part violates the above bound. For $M_t = 165$ GeV there is a small region for which $M_{1/2}$ is rather small, m_0 is rather large and for which this condition is fulfilled (see Fig. 2). For negative values of μ, the off–diagonal terms are small and with a very weak dependence on the soft breaking parameters m_0 and $M_{1/2}$. Hence, one obtains a bound which is basically equivalent to the positivity of the diagonal term,

$$m_U^2 > -(m_t^2 + D_{t_R}). \tag{36}$$

15

In the present case, Eq.(36) is equivalent to

$$m_0^2 < 6 \left(m_t^2 + D_{t_R} + 4M_{1/2}^2 \right). \tag{37}$$

Quite generally, the condition of absence of color breaking, derived from Eq. (32) assures the fulfillment of Eq.(36) since the actual bound coming from the absence of color breaking is stronger than the one implied by Eq. (37).

Fig. 2 shows the dependence of the lightest stop mass on the lightest chargino mass for case I of non–universal mass parameters at M_{GUT} and four different values of the top quark mass. For low values of $M_t \leq 160$ GeV, light charginos are not allowed. This is due to the Higgs bounds and the impossibility of getting large radiative corrections due to the bounds on m_0 derived from constraints in the stop sector and the absence of a color breaking minimum. For $M_t \simeq 165$ GeV, there is a regime with light charginos and $\mu > 0$, for which Eq. (35) and the Higgs mass bounds are fulfilled. Apart from this region, light charginos do not appear in the spectrum for this low values of $\tan \beta$. For $M_t \geq 175$ GeV, there is a clear distinction between positive values of μ (lower $m_{\tilde{t}_1}$) and negative values of μ (larger $m_{\tilde{t}_1}$). As can be seen from figure 5, for negative values of μ and $M_t < 185$ GeV, light charginos are not allowed, due to the constraints in the Higgs sector.

Finally, for the last condition under study, for which $m_{H_2}^2(0) = 0$ and $m_{H_1}^2(0) = 2m_Q^2(0) \equiv m_0^2$ (case II), the requirement of absence of tachyons in the stop sector differs from the other two cases only in the limit of large values of the soft supersymmetry breaking terms for the scalar fields at the grand unification scale. Since now both m_Q^2 and m_U^2 grow for large values of m_0^2, low values of the top squark masses may only be achieved for large values of the left–right mixing, which naturally arise in the low $\tan \beta$ regime. In particular, for $\tan \beta \simeq 1.3$, which approximately corresponds to $M_t \simeq 165$ GeV, and low values of the common gaugino mass $M_{1/2}$, in order to avoid problems in the stop spectrum, it is necessary to have $m_0^2 \geq 0.2m_t^2$. This bound becomes stronger for lower values of $\tan \beta$. On the contrary, for large values of the top quark mass, $M_t \geq 175$ GeV, no bound on m_0 is obtained from these considerations. Figure 3 shows the dependence of the lightest stop quark mass on the lightest chargino mass for case II and four different top quark mass values. For low values of $M_t \leq 165$ GeV, the colour breaking constraints are sufficiently strong to put restrictions on large values of m_0, particularly for low values of the chargino mass ($M_{1/2}$) and positive values of μ. For larger values of M_t, there is again a distinction between positive and negative values of μ. Observe that, due to the larger mixing, lower values of the lightest stop are always more easily obtained for positive values of μ.

5.2   Higgs Spectrum

Other important features of the spectrum at the infrared fixed point are associated with the Higgs sector. The Higgs spectrum is composed by three neutral scalar states –two CP-even,

h and H, and one CP-odd, A, and two charged scalar states H$^{\pm}$. Considering the one loop leading order corrections to the running of the quartic couplings –those proportional to m_t^4– and neglecting in a first approximation the squark mixing, the masses of the scalar states are given by,

$$m_{h,H}^2 = \frac{1}{2}\left[m_A^2 + M_Z^2 + \omega_t \right.$$
$$\left. \pm\sqrt{(m_A^2 + M_Z^2)^2 + \omega_t^2 - 4m_A^2 M_Z^2 \cos^2(2\beta) + 2\omega_t \cos(2\beta)(m_A^2 - M_Z^2)}\right] \quad (38)$$

$$m_A^2 = m_1^2 + m_2^2 + \frac{\omega_t}{2}$$
$$= \left[m_{H_1}^2(0) + \left(\frac{2m_Q^2(0) - m_{H_2}^2(0)}{2}\right) + 4M_{1/2}^2 - \frac{\omega_t}{2}\right]\frac{(1 + \tan^2\beta)}{(\tan^2\beta - 1)} \quad (39)$$

$$m_{H^{\pm}}^2 = m_A^2 + M_W^2 . \quad (40)$$

In the above, we have omitted the one loop contributions proportional to ω_t/m_t^2, since for $\tan\beta > 1$ they are negligible with respect to the other contributions. From Eq. (39) it follows that, for lower values of $\tan\beta$, the value of the CP-odd eigenstate mass is enhanced. Moreover, larger values of m_A, implies as well that the charged Higgs and the heaviest CP-even Higgs becomes heavier in such regime. Indeed, for low values of $\tan\beta \leq 2$ ($M_t \leq 190$ GeV) and for the experimentally allowed range for the other mass parameters, the CP-odd Higgs is always heavier than 150 GeV. In this regime, the radiative corrections give only a relevant contribution to the lightest CP–even Higgs mass, Eq. (38). In fact, for these large values of m_A, m_h acquires values close to its upper bound, which is independent of the exact value of the CP–odd mass [45]–[47]:

$$(m_h^{max})^2 = M_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\left[\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \Delta_{\theta_{\tilde{t}}}\right] \quad (41)$$

In the above, we have now considered the expression in the case of non-negligible squark mixing $\Delta_{\theta_{\tilde{t}}}$ is a function which depends on the left-right mixing angle of the stop sector and it vanishes in the limit in which the two mass eigenstates are equal: $m_{\tilde{t}_1} = m_{\tilde{t}_2}$ [45]-[47],

$$\Delta_{\theta_{\tilde{t}}} = \left(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2\right)\frac{\sin^2 2\theta_{\tilde{t}}}{2m_t^2}\log\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right)$$
$$+ \left(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2\right)^2\left(\frac{\sin^2 2\theta_{\tilde{t}}}{4m_t^2}\right)^2\left[2 - \frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\log\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right)\right], \quad (42)$$

where $\theta_{\tilde{t}}$ is the stop mixing angle.

Furthermore, the infrared fixed point solution for the top quark mass has explicit important implications for the lightest Higgs mass. For a given value of the physical top quark

mass, the infrared fixed point solution is associated with the minimun value of $\tan\beta$ compatible with the perturbative consistency of the theory. For values of $\tan\beta \geq 1$, lower values of $\tan\beta$ correspond to lower values of the tree level lightest CP-even mass, $m_h^{tree} = M_Z|cos2\beta|$. Therefore, the infrared fixed point solution minimizes the tree level contribution and after the inclusion of the radiative corrections it still gives the lowest possible value of m_h for a fixed value of M_t [8], [21],[48], [49]. This property is very appealing, in particular, in relation to future Higgs searches at LEP2, as we shall show explicitly below. In figure 4 we present the values of the lightest Higgs mass as a function of the top quark mass at its infrared fixed point solution, for the case of universal boundary conditions, and performing a scanning over the mass parameters up to low energy squark masses of the order of 1 TeV. For comparison, we present the upper bounds on the Higgs mass which is obtained for larger values of $\tan\beta$. Observe that, for $M_t \leq 175$ GeV, there is approximately a difference of 30 GeV between the upper bound at and away from the top quark mass fixed point. As we shall discuss below in more detail, although the characteristic of the Higgs spectrum depend on the boundary conditions at the grand unification scale, these upper bounds have a more general validity.

In general, the lightest CP-even Higgs mass spectrum is a reflection of the characteristics of the stop spectrum presented in Figs. 1 - 3. For the same chargino mass, larger Higgs mass values are obtained for positive values of μ than for negative values of μ. Quite generally, the upper bound on the Higgs mass does not depend on the different structure of the boundary conditions of the scalar mass parameters at the grand unification scale. It reads $m_h \leq 90$ (105) (120) GeV, for $M_t \leq 165$ (175) (185) GeV. Fig. 5, 6 and 7 present the dependence of the lightest CP-even Higgs on the chargino mass for four different values of the top quark mass and for the case of universal scalar mass m_0 and for the cases of non–universal mass parameters I and II, respectively.

Both the universal case and case II present similar features and are almost indistinguishable from the point of view of the lightest CP–even Higgs spectrum (Figures 5 and 7). For $M_t \simeq 165$ GeV, the Higgs mass becomes larger for a chargino mass $m_{\tilde{\chi}_1} \simeq 100$ GeV, than for moderate values of the chargino mass. The Higgs mass becomes, however, tightly bounded from above and it is always in the regime to be tested at LEP2. For larger values of the top quark mass, $M_t \geq 175$ GeV, and for negative (positive) values of μ, the Higgs mass lies mostly within (beyond) the experimentally reachable regime. Observe that, even if a light chargino is observed at LEP2, $m_{\tilde{\chi}_1^+} < 90$ GeV, nothing guarantees the observation of the lightest CP–even Higgs, particularly for larger values of the top quark mass, $M_t \geq 175$ GeV.

Case I (Fig. 6) is easily distinguishable from the above two cases, due to the more definite values of the Higgs mass related to the smaller allowed dependence on the scalar mass parameter m_0^2. Observe that, although the absolute upper bound on m_h for a given M_t does not significantly change, due to this particular structure of the high energy soft supersymmetry breaking mass parameters, the Higgs mass is in general pushed to lower

values than in the case of universal parameters. Therefore, unlike these two cases, the observation of a light chargino at LEP2 would almost guarantee the observation of a light neutral Higgs if $M_t \leq 185$ GeV in this case.

6 Precision Data Variables

In this section we shall define the experimental variables, which we shall use to analyze the implications of the infrared fixed point solution for the precision data analysis at LEP. In particular, we will follow the procedure of Altarelli, Barbieri, Caravaglios and Jadach [52]-[54], which consist in parametrizing the electroweak radiative corrections in terms of four parameters: ϵ_1, which is directly related to the Z-boson lepton width and is closely related to the parameter $\Delta\rho(0)$ usually defined in the literature [55](see below), the parameter ϵ_2, which is related to the parameter Δr_W, which measures the radiative corrections to the $W^\pm$ boson masses [62], the parameter ϵ_3, closely related to the radiative corrections to the weak mixing angle, and the parameter ϵ_b, which is related to the radiative corrections to the Z-$b\bar{b}$ vertex [64]-[67]. In this work, we shall concentrate on the parameters ϵ_1 and ϵ_b which are the only ones which keep a quadratic dependence on the top quark mass. This parametrization is based on the precise knowledge of G_F, α and M_Z^2, which are used as a basis for the precision data analysis.

The variable ϵ_1 may be directly obtained from the measurements of the Z–boson width and the forward–backward lepton asymmetries. Indeed the forward backward asymmetries may be parametrized in terms of the renormalized vector and axial lepton–Z boson couplings, g_V and g_A in the following way:

$$A_{FB}^l = \frac{3(g_V/g_A)^2}{\left[1 + (g_V/g_A)^2\right]^2}.$$ (43)

From g_V/g_A is it possible to define an effective weak mixing angle

$$\begin{aligned} \frac{g_V}{g_A} &= 1 - 4\sin^2\theta_W^{eff} \\ &= 1 - 4\left(1 + \Delta k\right)s_0^2, \end{aligned}$$ (44)

where

$$s_0^2 c_0^2 = \frac{\pi\alpha(M_Z)}{\sqrt{2}G_F M_Z^2}.$$ (45)

Δk is a measure of the radiative corrections to the weak mixing angle, which are quadratically dependent on the top quark mass. Observe that the angle s_0^2 already contains the running between low energies and the energy scale M_Z.

The total leptonic width may be also parametrized in terms of the axial and vector lepton couplings,

$$\Gamma_l = \frac{GM_Z^3}{6\pi\sqrt{2}}g_A^2\left(1 + \frac{g_V^2}{g_A^2}\right) \tag{46}$$

From the knowledge of the asymmetris and the lepton width one can obtain the axial coupling

$$g_A^2 = \frac{1}{4}\left(1 + \Delta\rho\right). \tag{47}$$

Then, the variable $\epsilon_1 \equiv \Delta\rho$ receives four different contributions[56]:

$$\epsilon_1 = e_1 - e_5 - \frac{\delta G}{G} - 4\delta g_A, \tag{48}$$

where $e_1 \equiv \Delta\rho(0)$ is given by,

$$e_1 = \frac{\Pi_{33}(0) - \Pi_{WW}(0)}{M_W^2}, \tag{49}$$

with $\Pi_{33}(0)$ and $\Pi_{WW}(0)$ the zero momentum vacuum polarization contributions to the W_3 and $W^\pm$ gauge bosons. In general,

$$\Pi_{ij}^{\mu\nu}(q) = -ig^{\mu\nu}\Pi_{(}q^2) + q^\mu q^\nu terms. \tag{50}$$

with $i, j = W, \gamma, Z$ or $i, j = 0, 3$ for the W_3 or B bosons, respectively. The term e_5 proceeds from the wave function renormalization constant of the Z boson at $q^2 = M_Z^2$ and is given by

$$e_5 = \left\{q^2\left[\frac{d}{dq^2}\frac{(\Pi_{ZZ}(q^2) - \Pi_{ZZ}(0))}{q^2}\right]\right\}_{q^2=M_Z^2}. \tag{51}$$

The contribution of e_1 and e_5 include all the dominant vacuum polarization effects to the renormalized coupling g_A. Finally, the vertex and box corrections are included in the variables δg_A and $\delta G/G$, as described, for example, in Ref. [56]. The dominant contributions to the ϵ_1 parameter are described in Appendix A.

Using Eqs. (44)–(47) and the precise values for G_F, $\alpha(M_Z)$ and M_Z in the standard model, the variable ϵ_1 is related with the asymmetries and the Z–boson leptonic width through the following expression [57],

$$\epsilon_1 = -0.9882 + 0.01196\frac{\Gamma_l}{MeV} - 0.1511\frac{g_V}{g_A}. \tag{52}$$

Analogously to the variable ϵ_1, the variable ϵ_b may be defined as a function of the axial and vector couplings of the b–quark to the Z^0–boson. In the low $\tan\beta$ regime, the relevant contributions, quadratically dependent on the top quark mass, may be analysed in terms of

20

only the coupling of the left handed bottom quarks to the Z^0 gauge boson. Formally, in this regime ϵ_b is defined from the relation

$$g_A^b = -\frac{1}{2}\left(1 + \frac{\Delta\rho}{2}\right)(1 + \epsilon_b),\tag{53}$$

with

$$g_L^b = \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W^{eff} - \frac{\epsilon_b}{2}\right)\left(1 + \frac{\Delta\rho}{2}\right),\tag{54}$$

and

$$g_R^b = \frac{\sin^2\theta_W^{eff}}{3}\left(1 + \frac{\Delta\rho}{2}\right).\tag{55}$$

Experimentally, the variable ϵ_b can be best obtained from the ratio of the $Z \to b\bar{b}$ width to the total hadronic width. It can be shown that the branching ratio is given by [57]

$$\frac{\Gamma_b}{\Gamma_h} \simeq 0.2182\left[1 + 1.79\epsilon_b - 0.06\epsilon_1 + 0.07\epsilon_3\right],\tag{56}$$

where the variable ϵ_3 is defined as

$$\epsilon_3 = c_0^2\Delta\rho + \left(c_0^2 - s_0^2\right)\Delta k.\tag{57}$$

and depends only logarithmically on the top quark mass. The most relevant contributions to the variable ϵ_b in the low $\tan\beta$ regime and within the minimal supersymmetric standard model are described in Appendix B.

In the above, we have given the dependence of the precision data variables, on the observables which are most sensitive to it. From the point of view of the experimental analysis, however, it is possible to extend the fit of the variables ϵ_1, ϵ_3 and ϵ_b by the introduction of other measured observables. This may be performed by, for example, including all purely leptonic quantities at the Z^0–pole, or the data on the b–quark from the forward–backward asymmetry, or simply to include all observables measured at the Z^0 peak at the LEP experiment. This last step may be performed by assuming that all relevant deviations from the standard model may be associated with either vacuum polarization effects or corrections to the $Z \to b\bar{b}$ vertex. The global fit to the data reduces the dependence on any single experiment and hence provide a more realistic estimate of the precision data variables. For the comparison of the theoretical results to the experimental data, we shall use the values of the variables which are obtained from these extended fits at the 90 % confidence level.

7   Indirect Signals of Supersymmetric Particles

In this section we shall investigate the possible experimental signature of supersymmetric particles in the variables ϵ_1, ϵ_b and the rate of the rare b decay, $b \to s\gamma$. We shall study this

in the case of universality of the soft supersymmetry breaking parameters at the unification scale and in the two characteristic cases of non-universal soft supersymmetry breaking scalar mass parameters discussed in sections 3 and 4 (cases I and II). A related analysis, within the framework of superstring–inspired $SU(5) \times U(1)$ supergravity models, was recently performed in Ref. [68].

Before analysing each case in detail, let us summarize the most relevant supersymmetric effects in these three experimental variables. The main supersymmetric contributions to the variable ϵ_1 comes from the chargino and stop sectors and are summarized in Appendix A. The stop contribution is always positive, and becomes relevant whenever there are light stops, with masses $m_{\tilde{t}} < 300$ GeV and with a non-negligible component in the $\tilde{t}_L$ squark. Due to the renormalization group behaviour of the mass parameters m_Q^2 and m_U^2, m_Q^2 is always larger than m_U^2 at low energies (see Table 1 and Eq. (33)) and, hence, in the cases under analysis the light stop has a dominant right handed component. A left handed component appears mainly through the mixing, which does not strongly affect the behaviour of $\Delta\rho(0)$ [59]. Hence, even in the case of light stops, with masses lower than 100 GeV, the potentially large positive contributions to the ρ parameters are in general suppressed. Light charginos, instead, give a negative contribution to ϵ_1, which become large if the lightest chargino mass $m_{\tilde{\chi}_l^+} < 70$ GeV. Since in most cases, light stops may only appear when charginos with masses close to the present experimental bounds are present in the spectrum, the light stop effect is in general screened by the chargino contribution.

The main contributions to the variable ϵ_b in the minimal supersymmetric standard model in the low $\tan\beta$ regime come from the standard $W^\pm$–top loop, the charged Higgs–top and the chargino–stop loops and are summarized in Appendix B. The charged Higgs contribution pushes ϵ_b in the same direction as the standard model one, while the chargino contributions tend to suppress the standard model corrections to the $Z^0 - b\bar{b}$ vertex. In the models under consideration, for $M_t < 185$ GeV ($\tan\beta < 2$), the charged Higgs is sufficiently heavy to give only a moderate contribution to the ϵ_b variable. The chargino contribution, instead, may become sizeable, particularly when light charginos and light stops are present in the spectrum. The largest chargino contributions, quadratically dependent on the top quark mass, appear in the case in which the lightest stop has a relevant right handed component and the lightest chargino has a relevant component in the charged Higgsino. Although the first condition is mostly satisfied for the cases of soft supersymmetry breaking terms under consideration, due to the large values of μ, the lightest chargino has a dominant wino component, which reduces the supersymmetric effects on ϵ_b. Still, as we shall show, relatively large effects are still possible.

The decay rate $b \to s\gamma$ receives also contributions from the standard $W^\pm$–top loop, the charged Higgs–top loop and the chargino–stop loops. The predictions for this decay rate within the Standard Model has been recently analysed by several authors [69]. A general

expression for the supersymmetric contributions has been presented in Refs. [70] and [71], and we shall not rewrite it here. The relevant properties are the following: As in the case of ϵ_b, the charged Higgs contribution tends to enhance the standard model signal. In the supersymmetric limit, $\mu = 0$ and $\tan\beta = 1$, the stop-chargino contribution exactly cancels the charged and standard model ones and the total rate is zero. Although for the experimentally preferred values of $M_t \simeq 174 \pm 17$ GeV [51] cases under consideration, its infrared fixed point solution yields values of $\tan\beta$ close to one, the values of the sparticle masses are far away from their supersymmetric expressions. Indeed, large values of μ are obtained and the soft supersymmetry breaking terms are in general not negligible. Furthermore, in the cases analysed in this work, the supersymmetric contribution singles out the sign of the mass parameter μ. For moderate positive values of μ there is a large suppression of the standard model decay rate, while for moderate not negative values of μ the branching ratio tends to be enhanced (The dependence on the sign of μ is stronger in the large $\tan\beta$ regime, $\tan\beta \geq 30$ [69],[31], which will not be analysed in the present work). In addition, as we discussed section 5, for positive values of μ, due to the larger values of the stop mixing, it is easier to obtain smaller stop masses without being in conflict with the experimental bounds on the lightest CP-even Higgs mass ($m_h \geq 60$ GeV, for $m_A \geq 150$ GeV). For a given value of M_t ($\tan\beta$) larger values of μ are associated with heavy sparticles and hence the Standard Model decay rate tends to be recovered (see figure 12).

7.1 Dependence of the precision data variables on the light chargino mass

Figure 8 shows the dependence of the parameter ϵ_1 for the case of universal scalar masses at the grand unification scale and for three different values of the top quark mass. We observe that the qualitative features do not depend on the top quark mass: A departure from the Standard Model prediction occurs only for light chargino masses $m_{\tilde{\chi}_l^+} < 100$ GeV. As we discussed above, due to the small left handed component of the lightest stop, the main contribution is mainly negative, and this remain a general feature independently of the exact value of the top quark mass. Comparing the theoretical predictions with the recent fit to the LEP and SLD data [74],

$$\epsilon_1 = (3.5 \pm 2.9) \times 10^{-3} \tag{58}$$

at the 90 % confidence level (1.64 standard deviations), we see that, while light charginos with masses $m_{\tilde{\chi}_1^+} < 70$ GeV are not in conflict with the present data, only for large values of the top quark mass $M_t \geq 185$ GeV, are they preferred to heavier ones. On the other hand, very light charginos, with masses $m_{\tilde{\chi}_1} \leq 50$ GeV are disfavoured by the present data. It is important to remind the reader, however, that the present analysis looses its validity for

$m_{\tilde{\chi}_1} \leq 48$ GeV and hence the predictions for chargino masses lower than 50 GeV cannot be fully trusted.

For the case of non–universal soft supersymmetry breaking scalar mass parameters at the grand unification scale the main features of the universal case are preserved. In figures 9 and 10 we show the dependence of ϵ_1 on the chargino mass for the cases I and II, respectively, and a top quark mass $M_t = 175$ GeV. We see that, in spite of the quite different characteristics of the stop spectrum with respect to the universal case (see figures 1–3), no significant difference is observed with respect to the behaviour depicted in figure 8.

In figure 11 we present the behaviour of ϵ_b as a function of the lightest chargino mass for the case of universality of the soft supersymmetry breaking parameters and for three different values of the top quark mass. As in the case of ϵ_1, a significant departure from the Standard Model predictions may only be observed if the lightest chargino mass acquires rather small values, $m_{\tilde{\chi}_1} < 100$ GeV. However, in the presence of light charginos, the supersymmetric predictions for ϵ_b show a larger spreading of values for the precision data variable ϵ_b than for ϵ_1. This is related to the dependence of ϵ_b on the lightest stop mass. Indeed, due to the large component of the lightest stop on the right handed top squark, ϵ_b gets significantly changed for lower values of $m_{\tilde{t}_1}$. Since for $M_t \geq 175$ GeV, light stops are only possible for $\mu > 0$, the largest values of ϵ_b are associated with positive values of μ. Taking into account the recent fit to the variable ϵ_b [74],

$$\epsilon_b = (0.9 \pm 6.8) \times 10^{-3} \tag{59}$$

at the 90% confidence level, we see that the present data tends to prefer a light chargino, with mass $m_{\tilde{\chi}_1} \leq 80$ GeV. Observe that, for ϵ_b we take the fit to all LEP and SLD data, instead of taking the particular value obtained from the partial width Γ_b/Γ_h. If we just fit ϵ_b with this last variable according to the last reported data, $\Gamma_b/\Gamma_h = 0.2202 \pm 0.0020$ [58] we would get a larger central value, but also a larger error at the 90% confidence level, $\epsilon_b = (5.1 \pm 8.4) \; 10^{-3}$. Tighter bounds on the spectrum would be obtained if we took this latter value to perform our analysis.

Figure 10 shows the dependence of ϵ_b as a function of the lightest chargino mass for case II and a top quark mass $M_t = 175$ GeV. The characteristic features of this case are similar to the case of universal soft supersymmetry breaking parameters. Only a smaller concentration of points with larger values of ϵ_b is observed, related to the larger values of m_U^2 for the same value of the chargino mass parameters (see Table 1), which imply a smaller right handed component of the lightest stop.

Finally, in case I, larger values of the variable ϵ_b than in the other two cases may be obtained. In Figure 9 we display the corresponding dependence of ϵ_b as a function of the chargino mass for this case, with a top quark mass $M_t = 175$ GeV. Observe that values of ϵ_b close to zero are ppossible in this case. This is due to the fact that the right handed stop mass parameter m_U^2 can take very small values and the right handed component of the lightest stop

increases. In principle, a light stop may be obtained in this case for sufficiently low values of m_U^2, even when the mixing is negligible. However, large negative values of m_U^2 induce an unacceptable color breaking minimum, Eq. (32), and hence light stops and larger values of ϵ_b are only possible for positive values of μ, as in the universal case. Again, agreement of the theoretical results with the present experimental data at the 90 % confidence level may only be obtained for sufficiently light charginos $m_{\tilde{\chi}_1^+} < 70$ GeV.

8 On the $b \to s\gamma$ decay rate

In figure 12 we present the behaviour of the ratio of the prediction for the decay rate $b \to s\gamma$ to the standard model one, as a function of the supersymmetric mass parameter μ for the universal case and for three different values of the top quark mass M_t. As we discussed before, a clear dependence of $b \to s\gamma$ on the sign of μ is observed. For negative values of μ and a fixed value of the top quark mass $M_t \leq 175$ GeV, most of the theoretical predictions are close to the Standard Model ones, with a decay rate varying between 0.7 and 1.4 times the Standard Model prediction. The maximum departure is always noticed for the smallest values of $|\mu|$, associated with a light spectrum. For larger values of the top quark mas $M_t \geq 185$ GeV, a larger departure is possible, with a relative decay rate which may be close to two. Recently, the experimental value of the $b \to s\gamma$ decay branching ratio has been reported [72],

$$BR(b \to s\gamma) = (2.32 \pm 0.97) \times (1 \pm 0.15) \times [1 - (M_b - 4.87)] \times 10^{-4} \qquad (60)$$

where the second error is systematical, the bottom mass is given in GeV, and all errors have been treated at the 90% confidence level (1.64σ deviations). The above range allows, in principle, to put constraints in the supersymmetric spectrum. There are, however, large theoretical uncertainties associated with the standard model predictions, which for a top quark mass in the range $M_t \simeq 165$–185 GeV, and at the 90% confidence level reads [73],

$$BR(b \to s\gamma)(SM) \simeq (3.1 \pm 1.5) \times 10^{-4}, \qquad (61)$$

with a small dependence of the central value on the top quark mass ($\Delta BR(b \to s\gamma \simeq \pm 0.1\ 10^{-4}$), which is negligible in comparison to the theoretical error associated with QCD uncertainties. Hence, the presently allowed values for the relative decay rate at the 90 % confidence level translates into:

$$0.25 \leq \frac{BR(b \to s\gamma)}{BR(b \to s\gamma)(SM)} \leq 2.5, \qquad (62)$$

Observe that, to obtain the allowed range, we have minimized the theoretical uncertainty related to the bottom mass ($M_b = 4.9 \pm 0.3$ GeV) [35]. Had we included this uncertainty,

the range would be slightly larger than the one considered above. We believe, however, that the above gives a conservative estimate of the experimental values allowed at present, and it agrees quantitatively level with the one reported in Ref. [72]. Hence, the relatively large values of the decay rate obtained for $M_t \simeq 185$ GeV are still acceptable when all uncertainties are taken into account.

For positive values of μ, instead, the supersymmetric model tends to predict values of the decay rate smaller than in the Standard Model. The lower values of the stop mass associated with positive values of μ (and hence, with a larger mixing) contribute to this behaviour, since they enhance the negative chargino–stop loop contributions. In fact, for $M_t \leq 175$ GeV, both stops and charginos may be sufficiently light and the $b \to s\gamma$ decay rate may acquire very low values. As in the case of negative values of μ, however, apart from a few solutions for $M_t \simeq 165$ GeV, the present uncertainties do not allow to put strong bounds on these models for any of the values of M_t considered in Fig. 12.

For the case of non–universal parameters at the grand unification scale, cases I and II, the qualitative behaviour is the same as in the case of universal soft supersymmetry breaking parameters. In figures 9 and 10 we present the results for the relative decay rate as a function of μ for cases I and II, respectively, and a top quark mass $M_t = 175$ GeV. In case I lower values of the relative decay rate than in the universal case are possible for positive values of μ, and some of the predictions lie outside the experimentally allowed range. Due to the weak dependence of μ on m_0, μ is strongly correlated with the lightest chargino mass in this case, and hence, the solutions, which are experimentally excluded by these considerations, correspond to very light chargino mass values. As we shall see in the section 9, these are just the solutions which tend to give larger values of ϵ_b. In case II, instead, the theoretical predicted range is similar to the one predicted in the case of universal mass parameters, and $b \to s\gamma$ remains in the experimentally allowed range for all acceptable values of μ.

9 Correlated fit to the Data

In section 8, we present the theoretical predictions for different experimental variables as a function of relevant supersymmetric mass parameters. However, we did not discuss the correlations between the different variables, which become essential at the point of considering the experimentally allowed models. For instance, models with a value of ϵ_b closer to the present experimental central value may be in conflict with either the bounds on $b \to s\gamma$ or, since they are always obtained in the presence of light charginos, they may be in conflict with the present bounds on the ϵ_1 variable. It is the purpose of this section to analyze these correlations.

In figure 13 we give the correlation between ϵ_b and ϵ_1 for the case of universal mass

parameters and for three different values of the top quark mass M_t. We see that larger values of ϵ_b are necessarily associated with relatively smaller values of ϵ_1, although for $M_t \leq 175$ GeV there are a few solutions for which ϵ_1 remains at moderate values ($\epsilon_1 \simeq 1\text{--}2 \times 10^{-3}$) and ϵ_b is relatively large ($\epsilon_b \simeq -3 \times 10^{-3}$). These solutions are associated with light stops ($m_{\tilde{t}_1} < 150$ GeV) and light charginos ($m_{\tilde{\chi}_1} < 70$ GeV), which are not too close to the Z^0 boson mass threshold. For $M_t \geq 185$ GeV, stops are heavy and all solutions lie beyond the present 90% confidence level for ϵ_b. In fact, not only the standard model prediction further decreases with respect to lower top quark masses, but also the deviations with respect to the standard model prediction are smaller in this case. The variable ϵ_1, instead, can vary within a large range of values, depending on the lightest chargino mass.

In figure 14 we show the correlation between ϵ_b and ϵ_1 for cases I and II and for a top quark mass $M_t = 175$ GeV. Most of the properties of the case with universal mass parameters are preserved in these two cases. However, for acceptable values of ϵ_1, larger values of ϵ_b may be obtained in case I, while in case II smaller values of ϵ_b predicteded. These properties may be easily understood from the characteristics of the stop and chargino spectra shown in figures 1–3. Observe that, values of $\epsilon_b \simeq -2 \ 10^{-3}$ may be obtained in case I for acceptable values of $\epsilon_1 \simeq 1\text{--}2 \ 10^{-3}$. Observe that due to the behaviour of ϵ_1 for chargino masses $m_{\tilde{\chi}_1^+}$ very close to their production threshold at the Z^0 peak (see figure 8), our scanning shows few solutions in figure 14 for values of $\epsilon_1 \leq 2 \times 10^{-3}$. To fill that area with solutions woud demand a very fine scanning for values of $m_{\tilde{\chi}^+} < 60$ GeV.

Also interesting is the correlation between ϵ_b and $b \to s\gamma$, which we depict in figure 15 for the case of universal mass paramters at M_{GUT} and three values of the top quark mass. For negative values of μ (see also figure 12), larger values of ϵ_b are only possible for $M_t \leq 165$ GeV, for which perfectly acceptable values of $b \to s\gamma$ are obtained. Observe, however, that for $M_t \simeq 165$ GeV, the combination of the bounds on ϵ_b, ϵ_1 and $b \to s\gamma$ restricts $\epsilon_b < -3.2 \times 10^{-3}$ in this case. For $M_t \simeq 175$ GeV, $b \to s\gamma$ does not impose additional constraint but the bounds on ϵ_1 are strong enough to constraint $\epsilon_b < -3.6 \times 10^{-3}$ in this case. Much smaller values of ϵ_b are predicted for $M_t \geq 185$ GeV.

Figure 16 shows the correlation of $b \to s\gamma$ with ϵ_b for the cases of non–universal mass parameters I and II and a top quark mass $M_t = 175$ GeV. We see that, unlike the case of universal mass parameters, in case I the experimental range for $b \to s\gamma$ puts additional constraints on the spectrum. The variable ϵ_b can still take values lower than in the standard model, but still away from zero. For $M_t \simeq 175$ GeV, the correlated fit leads to a value of $\epsilon_b < -2.5 \ 10^{-3}$. As in the case of universal mass parameters at M_{GUT}, no significant variation of this bound is obtained for lower values of the top quark mass, while for larger values of the top quark mass ϵ_b tends to lower values. Finally, from the point of view of the range of allowed values for the experimental variables, case II is equivalent to the case of universal mass parameters, once the full experimental constraints considered in this work

are taken into account.

10　Conclusions

In the present work, we have analysed the theoretical predictions for the Higgs and supersymmetric spectrum and their indirect experimental signals at the top quark mass infrared fixed point solution for different boundary conditions of the scalar mass parameters at the grand unification scale. We have shown that even though the stop mass range significantly changes for different boundary conditions, the predicted lightest CP-even Higgs mass range remains unchanged, leading to rather general upper bounds for this mass, $m_h \leq 90$ (105) (120) GeV for $M_t \leq 165$ (175) (185) GeV. The correlation between the lightest Higgs mass and the chargino spectrum, however, depends on the chosen high energy boundary conditions for the mass parameters. Interesting enough, for $M_t \geq 175$ GeV, the observation of a light chargino at LEP2, does not guarantee the observation of the lightest CP-even Higgs mass, particularly for positive signs of μ for which the mixing is maximized. However, for $M_t < 185$ GeV, light stops may appear in the spectrum in this case. The allowed stop spectrum in the presence of a light chargino, strongly depends on the high energy boundary conditions. For two of the cases considered, the case of universal scalar mass parameters at M_{GUT} and the case I, for which the dominant dependence of the supersymmetric mass parameter μ on the scalar mass parameters vanishes, and a top quark mass $M_t \leq 175$ GeV, a light chargino, with mass $m_{\tilde{\chi}_1} \leq 70$ GeV is always associated with a light stop, with mass $m_{\tilde{t}_1} \leq 150$ GeV. In case II, for which the right handed stop mass parameter increases with the supersymmetry breaking scalar mass parameter m_0, heavier stops may appear together with light charginos.

The experimental variables analysed in this work are a reflection of the characteristics of the Higgs and supersymmetric spectrum. The variable ϵ_1 receives a significant negative correction only for low values of the chargino mass $m_{\tilde{\chi}_1^+} \leq 70$ GeV. The potentially large positive correction associated with the stop spectrum is mostly suppressed due to the relatively small left handed component of the lightest stop. These properties do not strongly depend on the different boundary conditions analysed in the present work. The variable ϵ_b receives also a significant correction, with respect to the Standard Model prediction only for sufficiently light charginos, $m_{\tilde{\chi}_1^+} < 100$ GeV. The correction is mainly positive, rendering ϵ_b closer to the experimentally allowed range than in the Standard Model case. Due to the large component of the lightest stop on the right handed top squark, the variable ϵ_b depends also on the lightest stop mass. Hence, it is mostly larger for positive values of μ, for which lighter stops are possible, particularly for $M_t \geq 175$ GeV. Finally, the corrections to the decay rate $b \to s\gamma$ are also maximized in the case of light charginos and light stops. This experimental variable has a strong dependence on the sign of μ. For positive values of μ, the prediction for the decay rate is generally larger than the standard model one, while for

negative values of μ it is generally smaller.

In the case of universal soft supersymmetry breaking scalar mass parameters, values of $\epsilon_b \simeq -2\ 10^{-3}$ may be obtained for sufficiently low values of the chargino masses. However, these values are achieved for very low values of the chargino masses and are in conflict with the experimental value of the variable ϵ_1 at the 90 % confidence level. Due to the present theoretical uncertainties in the computation of $BR(b \to s\gamma)$, the recent experimental measurement of this branching ratio yields no relevant additional constraints on the allowed mass parameters in the case of universal mass parameters at M_{GUT}. In general, for $M_t \geq 165$ GeV, the allowed values for the variable $\epsilon_b < -3.2 \times 10^{-3}$ in this case. Values compatible with the present experimental bounds on ϵ_b at the 90 % confidence level are always associated with light charginos $m_{\tilde{\chi}_1} < 100$ GeV and values of the variable ϵ_1 which are lower than the standard model prediction, but are mostly consistent with the present experimental data. In fact, the theoretical predictions for ϵ_1, within the experimentally allowed range for all variables, reads $\epsilon_1 \simeq 0.6$–$5\ 10^{-3}$. The decay rate $b \to s\gamma$ stays in the experimentally acceptable range, with values which tend to be mostly lower than in the Standard Model case.

In the case $m_{H_1}^2(0) = 0$, $m_{H_2}^2(0) = 2m_Q^2(0)$ (case I), many of the above discussed features are preserved, although larger values of ϵ_b are possible. Values of $\epsilon_b \simeq 0$, which are not in conflict with the bounds on the spectrum, lead however to too low values of either ϵ_1 or the branching ratio $BR(b \to s\gamma)$. In general, for $M_t \geq 165$ GeV, $\epsilon_b < -2.5\ 10^{-3}$ in this case. As in the universal case, consistency with the present experimental bounds lead to light charginos, values of the variable $\epsilon_1 \simeq 0.6$–5×10^{-3} and a $b \to s\gamma$ decay rate, which is mostly below the standard model prediction. Finally, in the case $m_{H_2}^2(0) = 0$, $m_{H_1}^2(0) = 2m_Q^2(0)$ (case II), the bounds on the ϵ parameters are equivalent to the ones found in the case of universal conditions at the grand unification scale.

The discrepancy between the experimentally allowed value of ϵ_b and the standard model prediction is mostly due to the lack of agreement of the standard model prediction for the branching ratio Γ_b/Γ_h and the corresponding experimental value. Indeed, the standard model prediction for ϵ_b lies beyond the experimental value at 90 % confidence level. The determination of this partial width is, however, a delicate experimental problem and there are some unresolved issues related to it. Hence, it is still premature to claim evidence of new physics based only on the ϵ_b variable. If the present tendency is mantained after these issues are solved, the low energy supersymmetric grand unified models have the power of closing the gap between theory and experiment. This will demand light charginos and light stops. If this is the case, we should see supersymmetric particles either at LEP2 or at the next Tevatron run. Hence, within the phenomenologically attractive scenario of minimal supersymmetric grand unified theories, if the present experimental bounds on ϵ_b were mantained, the above property, together with the tight upper bounds on the Higgs mass, promises a potentially

rich phenomenology at present and near future colliders.

Note added in proof After this work was completed, two independent works have appeared [75], in which the behaviour of the variable ϵ_b within the minimal supersymmetric model is analysed.

Appendix A.

In this appendix we describe the largest contributions to the parameter ϵ_1 in the minimal supersymmetric standard model. If charginos are sufficiently heavy, $m_{\tilde{\chi}_l^+} \geq 80 GeV$, the only large supersymmetric contributions to the parameter ϵ_1 comes from the stop–sbottom sector. This contribution is analogous to the dominant one coming from the top–bottom left handed multiplet, which reads,

$$\epsilon_1^{t-b} = \frac{3\,\alpha}{16\pi\,\sin^2\theta_W\,M_W^2}\left[M_t^2 + M_b^2 - \frac{2M_t^2 M_b^2}{M_t^2 - M_b^2}\ln\left(\frac{M_t^2}{M_b^2}\right)\right]. \tag{63}$$

Due to the large hierarchy between the top and the bottom masses, the above expression, Eq. (63) is completely dominated by the first term inside the bracket.

Concerning the stop–sbottom sector, in principle, only the supersymmetric partners of the left handed top and bottom quarks contribute to ϵ_1. However, due to the squark mixing governed by the A_t and μ parameters, these are not the mass eigenstates of the model. In terms of the mass eigenstates $m_{\tilde{t}_{1,2}}$ and $m_{\tilde{b}_{1,2}}$ the dominant stop - sbottom contribution to ϵ_1 is given by [62]

$$\begin{aligned}
\epsilon_1^{\tilde{t}-\tilde{b}} &= \frac{3\,\alpha}{16\pi\sin^2\theta_W\,M_W^2}\Big(T_{11}^2\,g(m_{\tilde{t}_1}, m_{\tilde{b}_1}) \\
&+ T_{12}^2\,g(m_{\tilde{t}_2}, m_{\tilde{b}_1}) - T_{11}^2\,T_{12}^2\,g(m_{\tilde{t}_1}, m_{\tilde{t}_2})\Big),
\end{aligned} \tag{64}$$

where T_{ij} is the mixing matrix which diagonalizes the stop mass matrix:

$$T\mathcal{M}_{st}T^\dagger = \mathcal{M}_{st}^D. \tag{65}$$

In the above, we have neglected the sbottom mixing angle, identifying $\tilde{b}_L \equiv \tilde{b}_1$; this is an excellent approximation for the low values of $\tan\beta$ we are considering. The function $g(m_1, m_2)$ is directly related to the dependence of the variable ϵ_1 on the top and bottom masses, Eq.(64),

$$g(m_1, m_2) = m_1^2 + m_2^2 - \frac{2m_1^2 m_2^2}{m_1^2 - m_2^2}\ln\left(\frac{m_1^2}{m_2^2}\right). \tag{66}$$

In the supersymmetric limit, $A_t = \mu = 0$, $\tan\beta = 1$, the squark mixing vanishes, and, the weak eigenstates become mass eigenstates with masses equal to their standard model partners. It is easy to verify that the contribution to the parameter ϵ_1 of the stop–sbottom sector becomes equal to the one of the top–bottom sector in this limit. On the other hand, for small mixing and a soft supersymmetry breaking parameter $m_Q^2 \gg m_t^2$,

$$\epsilon_1^{\tilde{t}-\tilde{b}} \simeq \epsilon_1^{t-b}\frac{m_t^2}{3m_Q^2}. \tag{67}$$

Hence, for sufficiently large values of the squark masses, the squark contribution to the ϵ_1 parameter vanishes.

The sleptons give a similar contribution to the parameter ϵ_1, although it is reduced by a factor 3 with respect to Eq. (64), due to the color factor. The only additional contribution that can become large is the chargino one, if their masses are close to the production threshold at the Z^0 peak, $m_{\tilde{\chi}_l} \simeq M_Z/2$. The derivative of the chargino vacuum polarization contribution goes to large values if the chargino masses approach the production threshold. Indeed, it behaves like

$$\Pi'(M_Z^2) \simeq \left(M_Z^2 - 4m_{\tilde{\chi}^+} \right)^{-1/2}. \tag{68}$$

The above expression formally diverges if charginos masses tend to $M_Z/2$. However, Eq. (51) losses its validity when $m_{\tilde{\chi}^+} - M_Z/2 < \Gamma_Z$, what means that it can only be trusted if the chargino masses are above 50 GeV [56]. In the following, we shall give here the dominant contribution to $\Pi'(M_Z^2)$ for sufficiently light charginos. The diagonalization of the chargino mass matrix is performed by a bi–unitary tranformation

$$U^* \mathcal{M}_{ch} V^\dagger = \mathcal{M}_D. \tag{69}$$

We can define the new matrices [60]

$$
\begin{aligned}
U_{ij}^L &= \frac{1}{2} U_{i2}^* U_{j2} - \cos^2 \theta_W \delta_{ij} \\
U_{ij}^R &= \frac{1}{2} V_{i2} V_{j2}^* - \cos^2 \theta_W \delta_{ij} \\
X_{ij} &= U_{ij}^L U_{ij}^{L*} + U_{ij}^R U_{ij}^{R*} \\
Y_{ij} &= U_{ij}^L U_{ij}^{R*} + U_{ij}^R U_{ij}^{L*}
\end{aligned} \tag{70}
$$

Then, the dominant (formally divergent in the limit $m_{\tilde{\chi}^+} \to M_Z/2$) chargino contributions to the ϵ_1 parameter are included in the definition of the variable e_5, Eq. (51), and are given by

$$
\begin{aligned}
e_5 &= \frac{2g_2^2}{\cos^2 \theta_W} \sum_{i,j} \Big\{ X_{ij} \Big[2M_Z^2 \left(B_{21}'(M_Z^2, M_i, m_j) \right. \\
&- \left. B_1'(M_Z^2, M_i, M_j) \right) + (M_j^2 - M_i^2) B_1'(M_Z^2, M_i, M_j) \\
&+ M_i(M_i X_{ij} - M_j Y_{ij}) B_0'(M_Z^2, M_i, M_j) \Big] \Big\},
\end{aligned} \tag{71}
$$

where B_i' simbolize the derivatives of the corresponding Passarino–Veltman function [63], which are given by

$$B_0'(M_Z^2, m_1^2, m_2^2) = \frac{1}{16\pi^2} \int_0^1 dx \, \frac{x(1-x)}{\chi(m_1^2, m_2^2, x)}$$

32

$$
\begin{aligned}
B_1'(M_Z^2, m_1^2, m_2^2) &= \frac{1}{16\pi^2} \int_0^1 dx \, \frac{x^2(1-x)}{\chi(m_1^2, m_2^2, x)} \\
B_{21}'(M_Z^2, m_1^2, m_2^2) &= \frac{1}{16\pi^2} \int_0^1 dx \, \frac{x^3(1-x)}{\chi(m_1^2, m_2^2, x)},
\end{aligned}
\tag{72}
$$

where

$$
\chi(m_1^2, m_2^2, x) = m_1^2 + (m_2^2 - m_1^2 - M_Z^2)x + M_Z^2 \, x^2
\tag{73}
$$

Observe that, for $m_1^2 = m_2^2$, the argument

$$
\chi(m_1^2, m_1^2, x) = M_Z^2 \left[(x - 1/2)^2 + (m_1^2/M_Z^2 - 1/4) \right],
\tag{74}
$$

and the derivative of the Passarino–Veltman functions listed above become hence singular for $m_1^2 \to M_Z^2/4$.

Appendix B.

In this appendix, we include the relevant formulae for the computation of the parameter ϵ_b in the minimal supersymmetric standard model for the low $\tan\beta$ regime. The main standard contribution come from the Standard top quark - W^+ one loop diagram. This may be expressed, within an excellent approximation for $M_t \geq 160$ GeV, as a series in the parameter $r = M_t^2/M_W^2$, namely [64]

$$
\begin{aligned}
\epsilon_b^{SM} &= -\frac{\alpha}{8\pi\sin^2\theta_W}\left[r + 2.88\log(r) - 6.716 + \frac{(8.368\log(r) - 3.408)}{r}\right.\\
&+ \left.\frac{(9.126\log(r) + 2.26)}{r^2} + \frac{(4.043\log(r) + 7.41)}{r^3}\right]
\end{aligned}
\tag{75}
$$

In the low $\tan\beta$ regime, the main contributions to the $Z - b\bar{b}$ vertex, associated with the Higgs and supersymmetric particles come from the charged Higgs contribution, which tends to enhance the Standard Model signal, and the one coming from the chargino–stop one loop contribution, which tends to reduce the Standard Model signal.

The charged Higgs contribution is given by [65]

$$
\epsilon_b^{H^+} = -\frac{\alpha}{2\pi\sin^2\theta_W}F_b^{H^+}
\tag{76}
$$

with

$$
\begin{aligned}
F_b^{H^+} &= \frac{M_t^2}{2M_W^2\tan^2\beta}\left[b_1(m_{H^+}, M_t, M_b^2)v_L^{(t)} + \left(\frac{M_Z^2}{\mu_R^2}c_6(m_{H^+}, M_t, M_t)\right.\right.\\
&- \left.c_0(m_{H^+}, M_t, M_t) - \frac{1}{2}\right)v_R^{(t)} + \frac{M_t^2}{\mu_R^2}\,c_2(m_{H^+}, M_t, M_t)v_L^{(t)}\\
&+ \left.c_0(M_t, m_{H^+}, m_{H^+})\left(\frac{1}{2} - \sin^2\theta_W\right)\right],
\end{aligned}
\tag{77}
$$

where μ_R is a renormalization scale, $v_L^{(t)} = 0.5 - 2\sin^2\theta_w/3$ and $v_R^{(t)} = -2\sin^2\theta_W/3$, and $b_1(a, b, c)$, $c_k(a, b, c)$ with $k = 0, 2, 6$ are the corresponding reduced Passarino–Veltman functions. Since $m_b^2 \ll M_Z^2, M_t^2$, they are well approximated by

$$
\begin{aligned}
b_1(m_1, m_2, 0) &= \int_0^1 dx\, x\log\left(\frac{m_1^2 x + m_2^2(1-x)}{\mu_R^2}\right)\\
c_0(m_1, m_2, m_3) &= \int_0^1 dx\,\left(\frac{\tilde{\chi}(x)\log[\tilde{\chi}(x)] - \tilde{\chi}(x) - b(x)\log[b(x)] + b(x)}{a(x)}\right)
\end{aligned}
$$

$$
\begin{aligned}
c_2(m_1, m_2, m_3) &= \int_0^1 dx\, \frac{\log\left(\tilde{\chi}(x)\right) - \log\left(b(x)\right)}{a(x)} \\
c_6(m_1, m_2, m_3) &= \int_0^1 dx\, x\frac{\log\left(\tilde{\chi}(x)\right) - \log\left(b(x)\right)}{a(x)},
\end{aligned}
\tag{78}
$$

and the arguments $a(x)$ and $b(x)$ are given by

$$
\begin{aligned}
a(x) &= \frac{m_3^2 - m_1^2 - xM_Z^2}{\mu_R^2} \\
b(x) &= \frac{m_1^2 + x\left(m_2^2 - m_1^2\right)}{\mu_R^2},
\end{aligned}
\tag{79}
$$

while $\tilde{\chi}(x) = \chi(m_3^2, m_2^2, x)/\mu_R^2$ and $\chi(m_3^2, m_2^2, x)$ has been defined in Eq.(73).

The chargino contribution takes a somewhat more complicated expression. It is given by [65]

$$
\epsilon_b^{\tilde{\chi}^+} = -\frac{\alpha}{2\pi \sin^2 \theta_W} \left(F_b^{\tilde{\chi}^+}(M_t) - F_b^{\tilde{\chi}^+}(0) \right),
\tag{80}
$$

where

$$
F_b^{\tilde{\chi}^+}(M_t) = F_b^{\tilde{\chi}^+(a)}(M_t) + F_b^{\tilde{\chi}^+(b)}(M_t) + F_b^{\tilde{\chi}^+(c)}(M_t),
\tag{81}
$$

and

$$
\begin{aligned}
F_b^{\tilde{\chi}^+(a)}(M_t) &= \sum_{i,j} b_1(m_{\tilde{t},j}, M_i, m_b^2) \left|\Lambda_{j,i}^L\right|^2, \\
F_b^{\tilde{\chi}^+(b)}(M_t) &= \sum_{i,j,k} c_0(M_k, m_{\tilde{t},i}, m_{\tilde{t},j}) \left(\frac{2}{3}\sin^2\theta_W \delta_{ij} - \frac{1}{2}T_{i1}^* T_{j1}\right) \Lambda_{ik}^L \Lambda_{jk}^{*L}, \\
F_b^{\tilde{\chi}^+(c)}(M_t) &= \sum_{i,j,k} \left\{ \left[\frac{M_Z^2}{\mu_R^2} c_6(m_{\tilde{t},k}, M_i, M_j) - \frac{1}{2} - c_0(m_{\tilde{t},k}, M_i, M_j)\right] U_{ij}^R \right. \\
&\quad \left. + \frac{M_i M_j}{\mu_R^2} c_2(m_{\tilde{t},k}, M_i, M_j) U_{ij}^L \right\} \Lambda_{ki}^L \Lambda_{kj}^{*L},
\end{aligned}
\tag{82}
$$

with

$$
\Lambda_{ij}^L = T_{i1} V_{j1}^* - \frac{M_t}{\sqrt{2}M_W \sin\beta} T_{i2} V_{j2}^*
\tag{83}
$$

and T_{ij} (V_{ij}, U_{ij}) is the stop (chargino) mixing mass matrix (matrices) defined in Appendix A. Observe that both the parameter Λ_{ij}^L and the squark mass parameters have a dependence on the top quark mass. Indeed, if the top quark mass were negligible, the squark mass parameters would acquire an approximately common value $m_{\tilde{t}}^2 \simeq m_0^2 + 7M_{1/2}^2$. The function $F_b^{\tilde{\chi}^+}(0)$ becomes, hence, independent of the stop mixing matrix (which is formally equal to the identity in the limit $M_t = 0$).

FIGURE CAPTIONS

Fig. 1. Lightest stop mass as a function of the lightest chargino mass, for the case of universal soft supersymmetry breaking parameters at the grand unification scale and four different values of the physical top quark mass $M_t = 160$, 165, 175 and 185 GeV.

Fig. 2. The same as figure 1, but for the case I of non–universality for the scalar mass parameters at M_{GUT} : $m^2_{H_1}(0) = 0$, $m^2_{H_2}(0) = 2m^2_Q(0)$.

Fig. 3. The same as figure 1, but for the case II of non–universality for the scalar mass parameters at M_{GUT} : $m^2_{H_1}(0) = 0$, $m^2_{H_2}(0) = 2m^2_Q(0)$.

Fig. 4. Lightest CP–even Higgs mass as a function of the physical top quark mass, for the values of $\tan\beta$, which for each value of M_t corresponds to the top quark mass infrared fixed point solution (crosses). Also shown in the figure is the upper bound on the Higgs mass as a function of the top quark mass for values of $\tan\beta \simeq 5$–10.

Fig. 5. Lightest CP–even Higgs mass as a function of the lightest chargino mass for the case of universal scalar mass parameters at M_{GUT} and for the same values of the physical top quark mass $M_t = 165, 175$ and 185 GeV.

Fig. 6. The same as figure 5 but for the case I of non–universality of the soft super-symmetry breaking parameters at M_{GUT}.

Fig. 7. The same as figure 4 but for the case II of non–universality of the soft super-symmetry breaking parameters at M_{GUT}.

Fig. 8. Dependence of the precision data variable ϵ_1 on the lightest chargino mass for the case of universal supersymmetry breaking scalar mass parameters at M_{GUT} and for three different values of the top quark mass: $M_t = 165, 175$, 185 GeV.

Fig. 9. Dependence of the variables ϵ_1, ϵ_b as a function of the lightest chargino mass and the ratio of the supersymmetric prediction for the branching ratio $BR(b \to s\gamma)$ to the standard model one, as a function of the supersymmetric mass parameter μ, for the case I of non–universality of the soft supersymmetry breaking parameters at M_{GUT} and a top quark mass $M_t = 175$ GeV.

Fig. 10. The same as Fig. 9 but for the case II of non–universality of the soft super-

symmetry breaking parameters at M_{GUT}.

Fig. 11. Dependence of the variable ϵ_b on the lightest chargino mass for the case of universal scalar mass parameters at M_{GUT} and for three different values of the top quark mass: $M_t = 165, 175$ and 185 GeV.

Fig. 12. Dependence of the ratio of the supersymmetric prediction for the branching ratio $BR(b \to s\gamma)$ to the Standard Model one, as a function of the supersymmetric mass parameter μ for the case of universal scalar mass parameters at M_{GUT} and three different values of the top quark mass: $M_t = 165, 175$ and 185 GeV.

Fig. 13. Correlation between the variables ϵ_1 and ϵ_b for the case of universality of the soft supersymmetry breaking parameters at M_{GUT} and three different values of the top quark mass: $M_t = 165, 175$ and 185 GeV.

Fig. 14. The same as Fig. 13, but for cases I and II of non–universality of the scalar mass parameters at M_{GUT} and a top quark mass $M_t = 175$ GeV.

Fig. 15. Correlation between the variables ϵ_b and the ratio of the supersymmetric prediction for the branching ratio $BR(b \to s\gamma)$ to the standard model one, for the case of universality of the soft supersymmetry breaking parameters at M_{GUT} and three different values of the top quark mass: $M_t = 165, 175$ and 185 GeV.

Fig. 16. The same as Fig. 15, but for the cases I and II of non–universality of the scalar mass parameters at M_{GUT} and a top quark mass $M_t = 175$ GeV.

References

[1] C. T. Hill, Phys. Rev. D24 (1981) 691;
C. T. Hill, C. N. Leung and S. Rao, Nucl. Phys. B262 (1985) 517.

[2] L. Alvarez Gaume, J. Polchinski and M.B. Wise, Nucl. Phys. B221 (1983) 495;
Bagger, S. Dimopoulos, E. Masso, Phys. Rev. Lett. 55 (1985) 920.

[3] M. Carena, T.E. Clark, C.E.M. Wagner, W.A. Bardeen and K. Sasaki, Nucl. Phys. B369 (1992) 33.

[4] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D24 (1981) 1681; S. Dimopoulos and H. Georgi, Nucl. Phys. B 193 (1981) 150; L. Ibañez and G.G. Ross, Phys. Lett. 105B (1981) 439.

[5] P. Langacker and N. Polonsky, Phys. Rev. D47 (1993) 4028; Phys. Rev. D49 (1994) 1454.

[6] M. Carena, S. Pokorski and C.E.M. Wagner, Nucl. Phys. B406 (1993) 59.

[7] See, for example, B.D. Wright, Univ. of Wisconsin report MAD/PH/812, March 1994.

[8] M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, Nucl. Phys. B419 (1994) 213.

[9] M. Carena and C. Wagner, CERN preprint CERN-TH.7320/94, to appear in the proceedings of the 2nd IFT Workshop on Yukawa Couplings and the Origins of Mass, Gainesville, Florida, Feb. 1994, P. Ramond ed. (1994).

[10] V. Barger, M.S. Berger and P. Ohmann, Phys. Rev. D49 (1994) 4908.

[11] B. Ananthanarayan, K.S. Babu and Q. Shafi, Bartol Report BA-94-03 (1994).

[12] G.L. Kane, C. Kolda, L. Roszkowski and J.D. Wells, Michigan Report UM-TH-94-03 (1994).

[13] G.K. Leontaris and N.D. Tracas, Ioannina Report IOA.303/94 (1994).

[14] H. Baer et al. Florida Report FSU-HEP-940311 (1994).

[15] W. de Boer, R. Ehret and D.I. Kazakov, Univ. of Karlsruhe report IEKP-KA/94-07, May 1994.

[16] J. Gunion and H. Pois, UC Davis Report UCD-94-1 (1994).

[17] P. Nath and R. Arnowitt, CERN preprint CERN-TH.7288/94 (1994).

[18] H. Arason, D. J. Castaño, B. Keszthelyi, S. Mikaelian, E. J. Piard, P. Ramond and B. D. Wright, Phys. Rev. Lett. 67 (1991), 2933.

[19] S. Kelley, J.L. Lopez and D.V. Nanopoulos, Phys. Lett. B278 (1992) 140.

[20] S. Dimopoulos, L. Hall and S. Raby, Phys. Rev. Lett. 68 (1992) 1984, Phys. Rev. D45 (1992) 4192.

[21] V. Barger, M.S. Berger and P. Ohmann, Phys. Rev. D 47 (1993) 1093; V. Barger, M.S. Berger, P. Ohmann and R.J.N. Phillips, Phys. Lett. B314 (1993) 351.

[22] W. A. Bardeen, M. Carena, S. Pokorski and C. E. M. Wagner, Phys. Lett. B320 (1994) 110.

[23] L. Ibañez and C. Lopez, Nucl. Phys. B233 (1984) 511; L. Ibañez, C. Lopez and C. Muñoz, Nucl. Phys. B256 (1985) 218.

[24] J.P. Derendinger and C.A. Savoy, Nucl. Phys. B237 (1984) 307.

[25] A. Bouquet, J. Kaplan and C.A. Savoy, Nucl. Phys. B262 (1985) 299.

[26] See, for example, M. Olechowski and S. Pokorski, Phys. Lett. B214 (1988) 393;
B. Anantharayan, G. Lazarides and Q. Shafi, Phys. Rev. D44 (1991) 1613;
W. Majerotto and B. Mösslacher, Z. Phys. C48 (1990) 273;
D. Kapetanakis, M. Mondragon and G. Zoupanos, Zeit. F. Phys. C60 (1993) 181.

[27] R. Barbieri and G. Giudice, Nucl. Phys. B306 (1988) 63.

[28] R. Arnowitt and P. Nath, Phys. Rev. Lett. 69 (1992) 1014; Phys. Lett. B287 (1992) 89; B289 (1992) 368.

[29] L.J. Hall, R. Rattazzi and U. Sarid, preprint LBL–33997, June 1993.

[30] R. Hempfling, preprint DESY-93-092, July 1993.

[31] M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, CERN preprint, CERN-TH.7163/94, to appear in Nucl. Phys. B.

[32] R. Rattazzi, U. Sarid and L.J. Hall, Stanford Univ. preprint SU-ITP-94/15, to appear in the proceedings of the second IFT Workshop on Yukawa couplings and the origins of mass, Gainesville, February 1994, P. Ramond ed.;
R. Hempfling, DESY preprint 94-078 (1994);
C. Wagner and M. Carena, CERN preprint CERN-TH.7321/94, to appear in the proceedings of the second IFT Workshop on Yukawa couplings and the origins of mass, Gainsville, February 1994, P. Ramond ed.;
M. Olechowski and S. Pokorski, Max Planck Institute preprint MPI-PhT/94-40.

[33] M. Carena and C. E. M. Wagner, in Ref. [32].

[34] Y. Kawamura, H. Murayama and M. Yamaguchi, LBL preprint LBL–35731 (1994);
N. Polonsky and A. Pomarol, University of Pennsylvania preprint UPR-0616-T (1994);
D. Mattalliotakis and H. P. Nilles, Max Planck Institute preprint MPI-PhT/94-39 (1994).

[35] K. Hikasa et al., Particle Data Group, Phys. Rev. D45 (1992).

[36] M. Carena, K. Sasaki and C. E. M. Wagner, Nucl. Phys. B381 (1992) 66.

[37] P. Chankowski, S. Pokorski and J. Rosiek, Phys. Lett. B281 (1992) 100.

[38] H. Haber and R. Hempfling, Phys. Rev. D48 (1993) 4280.

[39] P. Chankowski, Phys. Rev. D41 (1990) 2877.

[40] K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog. Theor. Phys. 67 (1982) 590.

[41] M. Olechowski and S. Pokorski, Nucl. Phys. B404 (1993) 509.

[42] J.M. Frere, D.R.T. Jones and S. Raby, Nucl. Phys. B222 (1983) 11;
L. Ibañez andd C. Lopez, Phys. Lett. 126B (1983) 54;
L. Alvarez Gaumé, J. Polchinski and M. Wise, Nucl. Phys. B221 (1983) 495;
J. Ellis, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. 121B (1983) 123;
C. Kounnas, A. Lahanas, D.V. Nanopoulos and M. Quirós, Nucl. Phys. B236 (1984) 438.

[43] J.L. Lopez, D.V. Nanopoulos and Z. Zichichi, CERN preprint CERN-TH.7296/94 (1994).

[44] M. Drees, M. Glück and K. Grassie, Phys. Lett. B157 (1985) 164;
See also J.F. Gunion, H.E. Haber and M. Sher, Nucl. Phys. B306 (1988) 1.

[45] P. Langacker and N. Polonsky, Pennsylvania Report No. UPR-0594 T (1994).

[46] G. Gamberini, G. Ridolfi and F. Zwirner, Nucl. Phys. B331 (1990) 331.

[47] J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. 262 (1991) 477,
J.L. Lopez and D.V. Nanopoulos, Phys. Lett. B266 (1991) 397.

[48] C.E.M. Wagner, MPI preprint MPI-PH-93-25, April 1993, to appear in Proc. of *Properties of SUSY Particles*, Erice, Italy, Sep. 1992.

[49] J. Casas, J.R. Espinosa, M. Quiros and A. Riotto, CERN preprint CERN-TH.7334/94 (1994).

[50] S. Ferrara, C. Kounnas and F. Zwirner, CERN preprint CERN-TH.7192/94.
C. Kounnas, I. Pavel and F. Zwirner, CERN preprint CERN-TH.7185/94

[51] F. Abe et al., CDF Collab. FERMILAB-PUB-94/097-E.

[52] G. Altarelli, R. Barbieri and S. Jadach, Nucl. Phys. B369 (1992) 3.

[53] G. Altarelli, R. Barbieri and F. Caravaglios, Nucl. Phys. B405 (1993) 3.

[54] G. Altarelli, R. Barbieri and F. Caravaglios, Phys. Lett. B314 (1993) 357.

[55] M. Veltman, Nucl. Phys. B123 (1987) 89;
M. Chanowitz et al., Phys. Lett. B78 (1978) 285.

[56] R. Barbieri, M. Frigeni and F. Caravaglios, Phys. Lett. B 279 (1992) 169.

[57] G. Altarelli, CERN preprint CERN-TH.7072/93 (1993).

[58] D. Schaile, talk given at the 27th International Conference on High Energy Physics, Glasgow, July 1994.

[59] R. Barbieri and L. Maiani, Nucl. Phys. B 224 (1983) 32.

[60] J. A. Grifols and J. Sola, Phys. Lett. B137 (1984) 257; Nucl. Phys. B253 (1985) 47.

[61] R. Barbieri, M. Frigeni, F. Giuliani and H.E. Haber, Nucl. Phys. B341 (1990) 309;
P. Gosdzinsky and J. Sola, Phys. Lett. 254B (1991) 139;
M. Drees, K. Hagiwara and A. Yamada, Phys. Rev. D45 (1992) 1725;
J. Ellis, G.L. Fogli and E. Lisi, Phys. Lett. B324 (1994) 173; CERN preprint CERN-TH.7261-94 (1994).

[62] For recent studies of Δr_W within the MSSM, see
D. Garcia and J. Sola, Mod. Phys. Lett. A9 (1994) 211;
P. Chankowski, A. Dabelstein, W. Hollik, W. Mösle, S. Pokorski and J. Rosiek, Nucl. Phys. B417 (1994) 101.

[63] G. Passarino and M. Veltman, Nucl. Phys. B160 (1979) 151;
M. Green and M. Veltman, Nucl. Phys. B169 (1980) 137.

[64] J. Bernabeu, A. Pich and A. Santamaria, Phys. Lett. B200 (1988) 569;
W. Beenaker and W.Hollik, Z. Phys. C40 (1988) 141;
F. Boudjema, A. Djouadi and C. Verzegnassi, Phys. Lett. B238 (1990) 423.

[65] A. Djouadi, G. girardi, C. Verzegnassi, W. Hollik and F. Renard, Nucl. Phys. B349 (1991) 48.

[66] M. Boulware and D. Finnel, Phys. Rev. D48 (1993) 3081.

[67] G. Bhattacharyya and A. Raychaudhuri, Phys. Rev. D47 (1993) 2014; Phys. Rev. D49 (1994) 1156.

[68] J.L. Lopez, D.V. Nanopoulos, G.T. Park and A. Zichichi, Phys. Rev. D49 (1994) 4835.

[69] F. M. Borzumati, preprint DESY 93-090, August 1993;
M. A. Diaz, Phys. Lett. B322 (1994) 207;
S. Bertolini and F. Vissani, preprint SISSA 40/94/EP, March 1994;
P. Nath and R. Arnowitt, CERN preprint CERN-TH.7214/94f, March 1994;
J.L. Lopez , D.V. Nanopoulos, X. Wang and A. Zichichi, CERN preprint CERN-TH.7335/94, June 1994.

[70] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. B353 (1991) 591.

[71] R. Barbieri and G. Giudice, Phys. Lett. B309 (1993) 86.

[72] Cleo Collaboration, preprint CLEO CONF 94-1, July 1994.

[73] A. Ali, C. Greub and T. Mannel, Proc. ECFA Workshop on B-meson Factory, eds. R. Aleksan and A. Ali, DESY (1993);
A.J. Buras, M. Misiak, M. Münz and S. Pokorski, Max Placnk Institute preprint MPI-Ph/93-77 (1993).

[74] See for example, G. Altarelli, CERN preprint, CERN-TH.7319/94, June 1994.

[75] J.E. Kim and G.T. Park, Seoul University preprint SNUTP 94-66, August 1994;
J.D. Wells, C. Kolda and G.L. Kane, University of Michigan preprint UM-TH-94-23, July 1994.